

SUM THEOREMS FOR TOPOLOGICAL SPACES

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This paper is a study of Sum Theorems for various classes of topological spaces. Specifically, suppose that X is a topological space and $\{F_\alpha\}$ is a cover of X such that each F_α belongs to some class Q of topological spaces. When can we assert that X is in Q ? We shall concentrate our attention on those cases where the elements of $\{F_\alpha\}$ are either all open or all closed and the collection $\{F_\alpha\}$ is a σ -locally finite cover of X .

Throughout this paper Q will denote a class of topological spaces; e.g., normal spaces, paracompact spaces, etc. Perhaps the best known Sum Theorem is the so called Locally Finite Sum Theorem, hereinafter denoted (Σ) .

(Σ): Let X be a topological space and let $\{F_\alpha\}$ be a locally finite closed cover of X such that each F_α is in Q . Then X is in Q .

It is known that (Σ) holds when Q is the class of regular spaces [14], normal spaces [13], collectionwise normal spaces [13], paracompact spaces [11], stratifiable spaces [3], or metrizable spaces [14]. In §5 we show that (Σ) also holds for pointwise paracompact spaces.

The main results of the paper are in §3. In that section we prove three Sum Theorems, each of which holds for any class of topological spaces which satisfies (Σ) and is hereditary with respect to closed subsets. These results illustrate the importance of (Σ) in our study of Sum Theorems.

In §4 we give an application of one of the Sum Theorems, namely a Subset Theorem for totally normal spaces. This theorem closely parallels the result in [9].

The reader is referred to the following papers for definitions: collectionwise normal [1]; paracompact [11]; point finite collection [12]; stratifiable [2]. A topological space X is *pointwise paracompact* if every open cover of X has a point finite open refinement. According to Dowker [5] a normal space X is *totally normal* if every open subset U of X can be written as a locally finite (in U) collection of open F_σ subsets of X .

2. Examples. In this section we discuss two examples which nullify several conjectures and in addition will serve as a guide in selecting appropriate hypotheses for the Sum Theorems appearing in §3.