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CONDITIONS FOR A MAPPING TO HAVE THE SLICING STRUCTURE PROPERTY

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Let $p: E \rightarrow B$ be a fibering in the sense of Serre. As is well known the fibering need not be a fibering in any stronger sense. However it is expected that if certain conditions are placed on E, p or B then p might be a fibration in a stronger sense. This paper gives such conditions.

The main result of this paper is:

THEOREM 1. Let p be an n-regular perfect map from a complete metric space (E, d) onto a locally equiconnected space B. If $\dim E \times B \leqq n$ then p has the slicing structure property (in particular p is a Hurewicz fibration).

The following definitions will be needed.

DEFINITION 1. A space X is *locally equiconnected* if for each point x, there exists a neighborhood U_x of x and a map

$$N: U_x \times U_x \times I \rightarrow X$$

satisfying $N(a, b, 0) = a$, $N(a, b, 1) = b$, and $N(a, a, t) = a$.

DEFINITION 2. A map p from E to B is n-regular if it is open and satisfies the following property: given any x in E and any neighborhood U of x there exists a neighborhood V of x such that if $f: S^m \rightarrow V \cap p^{-1}(y)$ for some $y \in B$ $(m \leqq n)$ then there exists

$$F: B^{m+1} \rightarrow U \cap p^{-1}(y)$$

which is an extension of f.

DEFINITION 3. A family $\mathscr{S}$ of sets of Y is equi-LC^n if for every $y \in S \in \mathscr{S}$ and every neighborhood U of y in Y there exists a neighborhood V of y such that for every $S \in \mathscr{S}$, every continuous image of an m-sphere $(m \leqq n)$ in $S \cap V$ is contractible in $S \cap U$.

Note 1. If $p: E \rightarrow B$ is n-regular then the collection $\{p^{-1}(b) \mid b \in B\}$ is equi-LC^n.

DEFINITION 4. A family $\mathscr{S}$ of sets of a metric space (Y, d) is *uniformly* equi-LC^n with respect to d if given $\varepsilon > 0$ there exists $\delta > 0$ such that if $f: S^m \rightarrow S \cap N(x, \delta)$ $(m \leqq n$ and $S \in \mathscr{S})$ then there exists $F: B^{m+1} \rightarrow S \cap N(x, \varepsilon)$ which is an extension of f.

DEFINITION 5. A map $p: E \rightarrow B$ has the *covering homotopy pro-*