PACIFIC JOURNAL OF MATHEMATICS
Vol. 30, No. 2, 1969

A NOTE ON CERTAIN DUAL SERIES EQUATIONS INVOLVING LAGUERRE POLYNOMIALS

H. M. SRIVASTAVA

In this paper an exact solution is obtained for the dual series equations

$$(1)\qquad \sum_{n=0}^{\infty}\frac{A_n}{\Gamma(\alpha+n+1)}L_n^{(\alpha)}(x)=f(x)\,,\qquad 0\leqq x<y\,,$$

$$(2)\qquad \sum_{n=0}^{\infty}\frac{A_n}{\Gamma(\alpha+\beta+n)}L_n^{(\sigma)}(x)=g(x)\,,\qquad y<x<\infty\,,$$

where $\alpha+\beta+1>\beta>1-m$, $\sigma+1>\alpha+\beta>0$, m **is a positive integer,**

$$L_n^{(\alpha)}(x)=\binom{\alpha+n}{n}\,{}_1F_1[-n;\alpha+1;x]\,,$$

is the Laguerre polynomial and $f(x)$ **and** $g(x)$ **are prescribed functions.**

The method used is a generalization of the multiplying factor technique employed by Lowndes [4] to solve a special case of the above equations when

$$\sigma=\alpha,\ A_n=\Gamma(\alpha+n+1)\Gamma(\alpha+\beta+n)C_n,\ \alpha+\beta>0\quad\text{and}\quad 1>\beta>0\,.$$

In another paper by the present author [5] equations (1) and (2) have been solved by considering separately the equations when (i) $g(x)\equiv 0$, (ii) $f(x)\equiv 0$, and reducing the problem in each case to that of solving an Abel integral equation. Indeed it is easy to verify that the solution obtained earlier [5] is in complete agreement with the one given in this paper.

2. The following results will be required in the analysis.

(i) The orthogonality relation for Laguerre polynomials given by [3, p. 292 (2)] and [3, p. 293 (3)]:

$$(3)\qquad \int_0^\infty e^{-x}x^\alpha L_m^{(\alpha)}(x)L_n^{(\alpha)}(x)dx=\frac{\Gamma(\alpha+n+1)}{n!}\delta_{mn},\ \alpha>-1\,,$$

where δ_{mn} is the Kronecker delta.

(ii) The formula (27), p. 190 of [2] in the form:

$$(4)\qquad \frac{d^m}{dx^m}\{x^{\alpha+m}L_n^{(\alpha+m)}(x)\}=\frac{\Gamma(\alpha+m+n+1)}{\Gamma(\alpha+n+1)}x^\alpha L_n^{(\alpha)}(x)\,.$$

(iii) The following forms of the known integrals [2, p. 191 (30)] and [3, p. 405 (20)]: