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SOME RENEWAL THEOREMS CONCERNING A SEQUENCE OF CORRELATED RANDOM VARIABLES

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Consider a sequence $\{x_n\}$, $n = 1, 2, \cdots$ of random variables. Let $F_n(x)$ be the distribution function of $S_n = \sum_{k=1}^{n} x_k$ and $H_n(x)$, the distribution function of $M_n = \max_{1 \leqq k \leqq n} S_k$. Here we study the asymptotic behaviour of

1.1 $$\sum_{n=1}^{\infty} a_n G_n(x) ,$$

where $G_n(x)$ is to mean either $F_n(x)$ or $H_n(x)$ (so that if a property holds for both $F_n(x)$ and $H_n(x)$ it holds for $G_n(x)$ and conversely) and $\{a_n\}$ a suitable positive term sequence, when $\{x_n\}$ form

(i) a sequence of dependent random variables such that the correlation between x_i and x_j is $\rho, i \neq j, i, j = 1, 2, \cdots,$ $0 < \rho < 1$, $E(x_i) = \mu_i$, $i = 1, 2, \cdots$ and

1.2 $$\lim_{n \to \infty} \frac{\mu_1 + \mu_2 + \cdots + \mu_n}{n^\alpha} = \mu, \alpha > 1, 0 < \mu < \infty$$

and

(ii) a sequence of identically distributed random variables with $E(x_i) = \mu$, $i = 1, 2, \cdots$ such that the correlation between x_i and x_j is $\rho_{ij} = \rho^{|i-j|}, i, j = 1,2, \cdots, 0 < \rho < 1$.

Suitable examples are worked out to illustrate the general theory.

Let $N(x)$ be the first value of n such that $S_n \geqq x$, $x > 0$. $N(x)$ is a random variable and let

1.3 $$H(x) = E\{N(x)\} .$$

$H(x)$ is called the renewal function and much research work has been done with reference to the study of the asymptotic behaviour of $H(x)$ as $x \to \infty$. Feller has shown that

1.4 $$\lim_{x \to \infty} H(x)/x = 1/\mu ,$$

when $\{x_n\}$ form a sequence of independent and identically distributed random variables with $\mu = E(x_n), 0 < \mu < \infty$, the limit being interpreted as zero when $\mu = \infty$. Blackwell has generalised the above, by considering the renewal process $N(x, h)$ which denotes the number of renewals occuring in the interval $(x, x + h]$. He has shown that, for any fixed $h, (h > 0)$, if