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CONCERNING CONTINUA NOT SEPARATED BY ANY NONAPOSYNDETIC SUBCONTINUUM

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Certain theorems that apply to compact, metric continua that are separated by none of their subcontinua can be generalized and strengthened in those continua that are separated by none of their nonaposyndetic subcontinua. For those of the former type, if the continuum is aposyndetic at a point, it is locally connected at the point. The same conclusion is possible if the continuum is not separated by any nonaposyndetic subcontinuum. Also, if a continuum is separated by no subcontinuum and cut by no point, it is a simple closed curve. A second result of this paper is to prove that if no nonaposyndetic subcontinuum separates and no point cuts the continuum, then it is a cyclically connected continuous curve; in fact this yields a characterization of hereditarily locally connected, cyclically connected continua.

A third theorem characterizes an hereditarily locally connected continuum as an aposyndetic continuum that is separated by no nonaposyndetic subcontinuum. This is a somewhat stronger result than the known equivalence of hereditary local connectedness and hereditary aposyndesis.

A *continuum* is a closed, connected point set and the theorems of this paper are true for those continua that are compact and metric. If x is a point in the continuum M, then the continuum is *aposyndetic at* x if for every point y in $M - x$, there exists an open set U and continuum H such that $x \in U \subset H \subset M - y$. If M is aposyndetic at x for each point x in M, then M is *aposyndetic*, and M is *nonaposyndetic* if there is a point x in M such that M is not aposyndetic at x. By this definition a degenerate continuum is an aposyndetic continuum. The set S in M is said to *separate* M if $M - S$ is not connected and is said to *cut* M if for some pair of points $x, y \in M - S$, every subcontinuum of M intersecting both x and y must also intersect S. If every pair of points in M is contained in some simple closed curve lying in M, then M is *cyclically connected*. The continuum M is *hereditarily locally connected* if M is locally connected and every subcontinuum of M is locally connected, and M is *hereditarily aposyndetic* if it as well as each of its subcontinua is aposyndetic. In what follows, a subcontinuum of M is aposyndetic or nonaposyndetic if, with the relative topology from M, it is aposyndetic or nonaposyndetic respectively.

Bing has proved that if a continuum that is separated by no subcontinuum is aposyndetic at a point, it is locally connected at the