PACIFIC JOURNAL OF MATHEMATICS
Vol. 31, No. 1, 1969

BOUNDARY BEHAVIOR OF RANDOM VALUED HEAT POLYNOMIAL EXPANSIONS

ROBERT B. HUGHES

This paper is concerned with random series of the form $\sum_{n=0}^{\infty} X_n(\omega)a_n v_n(x, t)$ where the X_n's are random variables, the a_n's are real numbers, and the v_n's are heat polynomials as introduced by P. C. Rosenbloom and D. V. Widder. The sequences $\{a_n\}$ are assumed to satisfy $\limsup_{n\to\infty} |a_n|^{2/n}(2n/e) = 1$ which implies $\sum_{n=0}^{\infty} a_n v_n(x, t)$ has $|t| < 1$ as its strip of convergence, i.e., it converges to a C^2-solution of the heat equation in $|t| < 1$ and does not converge everywhere in any larger open strip. Associated with each sequence $\{a_n\}$ is its classification number from $[0, 1]$ which indicates how rapidly a_n tends to zero. Assumptions are placed on the random variables which imply that for almost every ω the series $\sum_{n=0}^{\infty} X_n(\omega)a_n v_n(x, t)$ has $|t| < 1$ as its strip of convergence.

The main results of the paper are two theorems. The first states that if $\{a_n\}$ has its classification number in $[0, 1/2)$, then for almost every ω the lines $t = 1$ and $t = -1$ form the natural boundary for $\sum_{n=0}^{\infty} X_n(\omega)a_n v_n(x, t)$. The second is concerned with sequences having their classification numbers in $(1/2. 1]$. The conclusion implies that for almost every ω no interval of either of the lines $t = 1$ or $t = -1$ is part of the natural boundary for $\sum_{n=0}^{\infty} X_n(\omega)a_n v_n(x, t)$.

The present work had it original motivation in the study of the boundary behavior of random power series. These are series of the form $\sum_{n=0}^{\infty} a_n(\omega)z^n$ where the a_n's are complex valued random variables and z is a complex number. Reference [1] contains a history of results in this area. One of the early results which helped to motivate the first part of the proof of our Theorem 1 is due to Paley and Zygmund in a 1932 paper [see 6, p. 220]. In this theorem it is assumed that $\sum_{n=0}^{\infty} a_n z^n$ is an ordinary power series with a finite radius of convergence. Letting $\{\phi_n\}$ be the sequence of Rademacher functions, the conclusion is that for almost every ω in $[0, 1]$ the series $\sum_{n=0}^{\infty} \phi_n(\omega)a_n z^n$ has its circle of convergence as its natural boundary.

More recently [see 3] V. L. Shapiro has considered series of the form $\sum_{n=0}^{\infty} X_n(\omega)H_n(x)$ where the X_n's are random variables and

$$\sum_{n=0}^{\infty} H_n(x)$$

is the spherical harmonic representation of a harmonic function in the unit ball. The harmonic continuability across the boundary of the unit ball of the functions $\sum_{n=0}^{\infty} X_n(\omega)H_n(x)$ was investigated. This