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WEAKLY HYPERCENTRAL SUBGROUPS OF FINITE GROUPS

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In this article the study of generalized Frattini subgroups of finite groups, developed by J. C. Beidleman and T. K. Seo, is continued. We call a proper normal subgroup H of a finite group G, a special generalized Frattini subgroup of G provided that $G = N_G(A)$ for each normal subgroup L of G and each Hall subgroup A of L such that $G = HN_G(A)$. Z. Janko proved that a subnormal subgroup K of a finite group G is π-closed, π is a set of primes, whenever $K/(K \cap \phi(G))$ is π-closed, where $\phi(G)$ denotes the Frattini subgroup of G. We prove that a subnormal subgroup K of a finite group G is π-closed whenever $K/(K \cap H)$ is π-closed where H is a special generalized Frattini subgroup of G. From this result we prove that a proper normal subgroup H of a finite group G is a special generalized Frattini subgroup of G if and only if H is a weakly hypercentral subgroup of G.

The properties of weakly hypercentral subgroups were developed by R. Baer. We obtain some of Baer's results in a different manner by using special generalized Frattini subgroups, and we also extend some of the properties of $\phi(G)$ to the class of generalized Frattini subgroups.

Some examples of special generalized Frattini subgroups of a finite group G are the Frattini subgroup $\phi(G)$, the center $Z(G)$ of a nonabelian group G, and the intersection $L(G)$ of all self-normalizing maximal subgroups of a nonnilpotent group G.

2. Notation.

G will always denote a finite group.

$|G|$ denotes the order of G.

$|G\colon H|$ is the index of the subgroup H in G.

$H^x = x^{-1}Hx$ where $x \in G$ and $H \leqq G$.

$Z(G)$ is the center of G.

$Z^*(G)$ is the hypercenter of G (i.e., the terminal member of the upper central series of G).

G' denotes the commutator subgroup of G.

$D(G)$ denotes the hypercommutator of G (i.e., the terminal member of the lower central series of G).

$N_G(H)$ denotes the normalizer of H in G.

$\phi(G)$ is the Frattini subgroup of G.