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APPROXIMATION OF TRANSFORMATIONS WITH CONTINUOUS SPECTRUM

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In several recent papers a new approach has been developed in the theory of approximation of automorphisms. Using this approach, Katok and Stepin have developed a new method which is very powerful and which has enabled them to solve several problems which had remained open for some time. Among the results they obtained is a characterization of automorphisms which are not strongly mixing in terms of the speed with which they can be approximated. Counter-examples may be given to show that the speed of approximation cannot be used to characterize those automorphisms which have continuous spectrum. In this paper certain related concepts are developed which do make it possible to deal in general with automorphisms which have continuous spectrum, and to distinguish those which are not strongly mixing from those which are strongly mixing among them. Since it is well-known that automorphisms have continuous spectrum if and only if they are weakly mixing, the result serves to distinguish between strong and weak mixing.

2. Notation and preliminaries. Let $(X, \mathfrak{F}, \mu)$ be the unit interval, the Lebesgue sets and Lebesgue measure. A map T of X onto X is an automorphism of X if it is invertible and measure preserving, i.e., if $A \in \mathfrak{F}$ then $T^{-1}A$, $TA \in \mathfrak{F}$ and $\mu(A) = \mu(T^{-1}A) = \mu(TA)$. If $B \in \mathfrak{F}$ we say that a map T of B onto B is an automorphism of B if it is an automorphism with B regarded as a measure space, that is with respect to $(B, \mathfrak{F} \cap B, \mu_B)$ where μ_B is defined for measurable subsets C of B by setting $\mu_B(C) = \mu(C)/\mu(B)$. As usual all statements are understood to hold almost everywhere, and we'll omit this phrase.

LEMMA 2.1. [3]. *The set $\mathscr{U}$ of automorphisms of X is a topological group with respect to the weak topology, that is, the topology obtained by taking neighborhoods to be finite intersections of sets of the form* $\{S: S \in \mathscr{U},\ \mu(SE \triangle TE) < \varepsilon\}$, $E \in \mathfrak{F}$.

In what follows it will be convenient to let $0 \cdot B = \varnothing, 1 \cdot B = B$ for sets $B \in \mathfrak{F}$. In what follows also, the topological space we refer to is $\mathscr{U}$, equipped with the weak topology.

DEFINITION 2.1. We say that ξ is a tower if $\xi = \{C_i, i = 1, \cdots, q\}$, an ordered collection of pairwise disjoint measurable sets