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CONTRACTIONS OF FUNCTIONS AND THEIR FOURIER SERIES

To Professor U. N. Singh on his 49th birthday

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The object of the present paper is to define a new type of contraction, called 'Shrivel', of a function and to prove a theorem on the absolute convergence of its Fourier series. Our theorem is similar to a theorem of M. Kinukawa, but as it is shown in the end the two results are essentially different. The original results in this direction are due to A. Beurling and R. P. Boas.

According to Beurling [1] a function g is said to be a contraction of function f if $|g(x) - g(y)| \leqq A|f(x) - f(y)|$, for all x, y, where A is an absolute constant. We shall assume throughout that the functions f and g are each L – integrable in $(-\pi, \pi)$ and periodic with the period 2π. We shall further let

$$f(x) \sim \frac{1}{2}a_0 + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx)$$

and

$$g(x) \sim \frac{1}{2}c_0 + \sum_{k=}^{\infty}(c_k \cos kx + d_k \sin kx) .$$

Kinukawa [3] has proved the following

THEOREM 1. *Let f and g be each continuous and g be a contraction of f, or more generally let f, $g \in L_2$ and*

$$\int_{-\pi}^{\pi} |g(x+h) - g(x)|^2 dx \leqq \int_{-\pi}^{\pi} |f(x+h) - f(x)|^2\, dx, \text{ for all } h .$$

If

$$(1) \qquad \sum_{n=1}^{\infty} n^{-3\alpha/2}\left(\sum_{k=1}^{n} k^2 p_k^2\right)^{\alpha/2} < \infty$$

and

$$(2) \qquad \sum_{n=1}^{\infty} n^{-\alpha/2}\left(\sum_{k=n+1}^{\infty} p_k^2\right)^{\alpha/2} < \infty ,$$

where $p_k^\alpha = |a_k|^\alpha + |b_k|^\alpha$ and $0 < \alpha \leqq 2$, then

$$(3) \qquad \sum_{k=1}^{\infty}(|c_k|^\alpha + |d_k|^\alpha) < \infty .$$