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# GLEASON PARTS AND CHOQUET BOUNDARY POINTS IN CONVOLUTION MEASURE ALGEBRAS

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**Let $M$ be a semisimple convolution measure algebra with structure semigroup $S$. Then each complex homomorphism of $M$ is given by integrating a semicharacter on $S$. Gleason parts can be defined on $\hat{S}$, the set of semicharacters on $S$, by considering the function algebra obtained from the transforms of elements of $M$. We give a partial characterization of the parts of $\hat{S}$ utilizing only the functional values of the elements of $\hat{S}$. We then completely characterize the one point parts of $\hat{S}$ utilizing only the functional values of elements of $\hat{S}$.**

If $S$ is a locally compact topological semigroup, then the measure algebra $M(S)$ is a member of an abstract class of Banach algebras called convolution measure algebras by Taylor in [12]. Other examples include $L'(G)$ for a locally compact group $G$ and the Arens-Singer algebras introduced in [1]. The convolution measure algebras form an extremely large and diverse class of algebras. In fact, a large number of interesting function algebras can be described as completions, in the spectral norm, of convolution measure algebras.

Taylor's main theorem in [12] is the following: if $M$ is a commutative, semisimple convolution measure algebra, then $M$ may be embedded in the measure algebra $M(S)$ of a certain canonical compact semigroup $S$, in such a way that every complex homomorphism of $M$ is determined by a continuous semicharacter on $S$.

Taylor's theorem identifies the maximal ideal space $\Delta$ of $M$ as the set $\hat{S}$ of all semicharacters on a compact semigroup $S$. This gives $\Delta$ a considerable amount of structure not generally enjoyed by maximal ideal spaces. It is natural to try to use this additional structure to help identify such standard objects as the Shilov and Choquet boundaries and the Gleason parts of $\Delta$.

If $H = \{f \in \hat{S} = \Delta : | f |^2 = | f |\}$, then Taylor showed that every element of $\hat{S} \backslash H$ lies in an analytic disc in $\hat{S}$. Hence the Choquet boundary points and the one point parts all lie in $H$ and the closure, $\bar{H}$, of $H$ contains the Shilov boundary (c.f. [7, 11, 12]). However, these results are not sharp since there are trivial examples where $H$ contains points which are neither one point parts, Choquet boundary points, nor Shilov boundary points. In fact, $\ell_1(J)$, where $J$ is the additive semigroup of nonnegative integers, is such an example. Here $\hat{S}$ is the unit disc in the complex plane $C$ and $H = \{z : | z | = 1 \text{ or } 0\}$. The point 0 is in $H$ and also is in a nontrivial part, $(\{z : | z | < 1\})$, of $\hat{S}$.