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THE CLOSED PRIME SUBGROUPS OF CERTAIN ORDERED PERMUTATION GROUPS

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The group $G = A(\Omega)$ of all order-preserving permutations of a chain Ω becomes a lattice-ordered group when ordered pointwise, i.e., $f \leqq g$ if and only if $\beta f \leqq \beta g$ for all $\beta \in \Omega$. Lloyd showed that for each $\omega \in \Omega$, the stabilizer subgroup $G_\omega = \{g \in G \mid \omega g = \omega\}$ is a closed prime subgroup of G. Our main result (Theorem 11) states that besides G itself, these subgroups, together with the stabilizer subgroups of Dedekind cuts of Ω, comprise all of the closed prime subgroups of G.

Actually, G need not be all of $A(\Omega)$. In § 2, we use Lloyd's result to show that for depressible or complete subgroups G of $A(\Omega)$, all stabilizer subgroups are closed. We find in § 3 that *every* closed convex l-subgroup C of G is of the form $C = \{g \in G \mid \bar{\Delta} g = \bar{\Delta}\}$ for some collection $\bar{\Delta}$ of points and cuts of Ω. In § 4 we prove the main theorem (stated above) for depressible groups. Finally, this theorem is applied to the question of the extent to which the l-group G determines the chain Ω, which was first considered by Holland [4]; and to the determination of the l-automorphisms of G, considered by Lloyd [5].

2. Stabilizer subgroups. Let Ω be a chain. A permutation g of Ω is said to *preserve order* if $\alpha \leqq \beta$ implies $\alpha g \leqq \beta g$ for all $\alpha, \beta \in \Omega$. The group $A(\Omega)$ of all order-preserving permutations (o-permutations) of Ω, ordered pointwise, is a lattice-ordered group (l-group). $A(\Omega)$ is not assumed to be transitive. For elementary information about l-groups, see [1].

$\bar{\Omega}$ will denote the completion of Ω by Dedekind cuts (without end points unless these end points belong to Ω). We shall consider Ω to be a subchain of $\bar{\Omega}$. Each $g \in A(\Omega)$ can be extended to $\bar{\Omega}$ by defining $\bar{\omega} g \,(\bar{\omega} \in \bar{\Omega})$ to be $\sup \{\beta g \mid \beta \in \Omega, \beta \leqq \bar{\omega}\}$. Thus $A(\Omega)$ can be considered to be an l-subgroup of $A(\bar{\Omega})$, i.e., a subgroup which is also a sublattice.

Let G be an l-subgroup of $A(\Omega)$, and thus also of $A(\bar{\Omega})$. G, or more properly the pair (G, Ω), is called an *ordered permutation group*. Holland [3, Th. 2] showed that every abstract l-group L is l-isomorphic to such a G. A subgroup H of G is *convex* if $h_1 \leqq g \leqq h_2$, with $h_1, h_2 \in H$ and $g \in G$, implies $g \in H$. H is a *prime* subgroup of G if H is a convex l-subgroup of G and if $g_1 \wedge g_2 = 1$, with $g_1, g_2 \in G$, implies $g_1 \in H$ or $g_2 \in H$. For $\bar{\omega} \in \bar{\Omega}$, we define the *stabilizer subgroup* $G_{\bar{\omega}}$ to be

$$\{g \in G \mid \bar{\omega} g = \bar{\omega}\} ,$$