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OUTER GALOIS THEORY FOR SEPARABLE ALGEBRAS

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Let G be a finite group of automorphisms of a ring Λ which has identity element. Let C be the center of Λ, let Γ be the subring of G-invariant elements of Λ, and assume that C is a separable extension of $C \cap \Gamma$. In the first section of this paper, it is shown that every finite group of automorphisms of Λ over Γ is faithfully represented as a group of automorphisms of C by restriction if, and only if, $\Lambda = C \otimes_{C\cap\Gamma} \Gamma$. Moreover, suppose that $\Lambda = C \otimes_{C\cap\Gamma} \Gamma$ and Ω is a subring of Λ such that $\Gamma \subseteqq \Omega \subseteqq \Lambda$. Then there exists a finite group H of automorphisms of Λ such that Ω is the subring of H-invariant elements of Λ if, and only if, $C \cap \Omega$ is a separable extension of $C \cap \Gamma$ and $\Omega = (C \cap \Omega) \otimes_{C\cap\Gamma} \Gamma$.

Let R be a commutative ring with identity element; and assume now that Λ is a separable algebra over R and G is a finite group of automorphisms of the R-algebra Λ. In the second section of this paper, it is shown that C is the centralizer of Γ in Λ if, and only if, $\Lambda = C \otimes_{C\cap\Gamma} \Gamma$. Moreover, suppose that $\Lambda = C \otimes_{C\cap\Gamma} \Gamma$ and Ω is a subalgebra of Λ such that $\Gamma \subseteqq \Omega \subseteqq \Lambda$. Then there exists a finite group H of automorphisms of Λ such that Ω is the subalgebra of H-invariant elements of Λ if, and only if, Ω is a separable algebra over R.

These results are obtained without the assumption of no nontrivial idempotent elements of C, which is required for the Kanzaki-DeMeyer Galois theory of separable algebras. Moreover, these results extend the Villamayor-Zelinsky Galois theory of commutative rings in the same way that the results of Kanzaki and DeMeyer extend the Chase-Harrison-Rosenberg Galois theory of commutative rings.

1. Galois theory. Throughout this paper, ring will mean ring with identity element and subring of a ring will mean subring which contains the identity element of the ring. Let Γ be a subring of a ring Λ. Call Λ a projective Frobenius extension of Γ if Λ is a finitely generated, projective right Γ-module and there is a (Γ, Λ)-bimodule isomorphism of Λ onto $\mathrm{Hom}_\Gamma(\Lambda, \Gamma)$. Call Λ a separable extension of Γ if the (Λ, Λ)-bimodule epimorphism of $\Lambda \otimes_\Gamma \Lambda$ onto Λ, which is determined by the ring multiplication in Λ, splits. Equivalently, Λ is a separable extension of Γ if there exist a positive integer n and elements x_i, y_i of Λ, for $1 \leqq i \leqq n$, such that $\sum_{i=1}^n x_i y_i = 1$ and $\sum_{i=1}^n a x_i \otimes y_i = \sum_{i=1}^n x_i \otimes y_i a$ in $\Lambda \otimes_\Gamma \Lambda$ for every $a \in \Lambda$. Also, let M be a left Λ-module and let N be a Γ-submodule of M. A canonical Λ-module homomorphism ϕ of $\Lambda \otimes_\Gamma N$ into M is determined by the