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LATTICES WITH NO INTERVAL HOMOMORPHISMS

J. D. LAWSON

This paper arose from the following analogous questions: (1) Does a distributive topological lattice on a continuum admit sufficiently many continuous lattice homomorphisms onto the unit interval to separate points, and (2) does a topological semilattice on a continuum admit sufficiently many continuous semilattice homomorphisms onto the unit interval to separate points? Earlier investigations of topological lattices and semilattices have provided partial positive solutions. However, examples of an infinite-dimensional distributive lattice and a one-dimensional semilattice which admit only trivial homomorphisms into the interval are presented in this paper.

A topological lattice consists of a Hausdorff space L together with a pair of continuous lattice operations $\wedge, \vee: L \times L \rightarrow L$. A topological semilattice consists of a Hausdorff space S together with a continuous semilattice operation $\wedge: S \times S \rightarrow S$. In the theory of topological lattices and semilattices, the following problem, raised by Dyer and Shields in [8], has received considerable attention: Does a distributive topological lattice (a topological semilattice) on a continuum admit sufficiently many continuous lattice (semilattice) homomorphisms onto the unit interval $[0, 1]$ to separate points?

Anderson [2] has given an affirmative answer for finite-dimensional lattices; Davies [7] and Strauss [12] have made further contributions to the problem for the lattice case. The semilattice question has been answered affirmatively for finite-dimensional semilattices on Peano continua [11]. The purpose of this paper is to provide examples that show the answer is not yes in general. We give examples of an infinite-dimensional distributive lattice and a one-dimensional semilattice which admit only trivial homomorphisms into the interval.

Since the idempotents of an abelian topological semigroup form a semilattice, these examples have ramifications with regard to representations of such semigroups. In particular, Brown and Friedberg [6] have a range space for representations (or semicharacters) of a special class of compact abelian semigroups. These representations separate points if and only if the homomorphisms of the idempotents into the interval separate points.

1. Preliminaries.

Let S be a (lower) semilattice. If $A \subset S$, we define

$$L(A) = \{y \in S: y \leqq x \text{ for some } x \in A\}$$