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ON SECONDARY CHARACTERISTIC CLASSES IN COBORDISM THEORY

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This paper introduces into cobordism theory a new notion borrowed from ordinary cohomology theory. Specifically, let ξ be a $U(n)$-bundle over the CW-complex X. Let E and E_0 be the total spaces of the associated bundles whose fibers are respectively the unit disc $E^{2n} \subset C^n$ and the unit sphere $S^{2n-1} \subset C^n$. The classifying map for ξ gives rise to an element $U_\xi \in \Omega_U^{2n}(E,E_0)$. One defines the Thom isomorphism φ: $\Omega_U^q(X) \to \Omega_U^{q+2n}(E, E_0)$ by $\varphi(x) = (p^*x)U_\xi$ and Euler class, $e(\xi)$ of ξ, by $e(\xi) = p^{*-1}j^*(U_\xi)$. For each $\alpha = (\alpha_1, \alpha_2, \cdots)$, let $cf_\alpha(\xi) \in \Omega_U^{2|\alpha|}(X)$ be the Conner-Floyd Chern class of ξ, and S_α: $\Omega_U^q(X, Y) \to \Omega_U^{q+2|\alpha|}(X, Y)$ be the operation defined by Novikov. Then one has the relation, $S_\alpha(e(\xi)) = cf_\alpha(\xi) \cdot e(\xi)$. Now if ξ is a bundle such that $e(\xi) = 0$, then one can define a secondary characteristic class

$$\Sigma_\alpha(\xi) \in \Omega_U^*(X) \ \mathbf{mod}\ (S_\alpha - cf_\alpha(\xi))\Omega_U^*(X)$$

by using the above relation. The object of this paper is to study some of the properties of such secondary characteristic classes.

Secondary characteristic classes adapt particularly to the study of embedding and immersion problems. Massey and Peterson and Stein developed secondary characteristic classes in ordinary cohomology theory [4][7][8], and Lazarov has studied secondary characteristic classes in K-theory [3]. We hope the secondary characteristic classes given here, and the operations on cobordism, defined by Novikov, will have some applications on embedding and immersion problems.

The organization of the papers is as follows. In §1 we collect some results on cobordism theory and give the definition of secondary characteristic classes of cobordism theory. In §2 we give an example and carry out some computations of these characteristic classes.

1. Definition of secondary characteristic classes. Let ξ be a $U(n)$-bundle over the CW-complex X. Let E and E_0 be the total spaces of the associated bundles whose fibres are respectively the unit disc $E^{2n} \subset C^n$ and the unit sphere $S^{2n-1} \subset C^n$. Then the Thom complex is the quotient space E/E_0. In particular, if we take ξ to be the universal $U(n)$-bundle over $BU(n)$, then the resulting Thom complex $M(\xi)$ is written $MU(n)$. The sequence of spaces