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COHOMOLOGY OF NONASSOCIATIVE ALGEBRAS

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A cohomology theory is constructed for an arbitrary non-associative (not necessarily associative) algebra satisfying a set of identities, within which the associative and Lie theories are special cases.

1. Exactness of the fundamental sequence through H^3. Let T be a set of identities, $\mathscr{A}$ a T-algebra over a commutative ring K with unit, M a T-bimodule for $\mathscr{A}$. When T is clear we call M an $\mathscr{A}$-bimodule. Let $(U(\mathscr{A}), \lambda_{\mathscr{A}}, \rho_{\mathscr{A}})$ be the universal T-multiplication envelope of $\mathscr{A}$ with $\lambda_{\mathscr{A}}, \rho_{\mathscr{A}}$ the canonical maps. When $\lambda_{\mathscr{A}}, \rho_{\mathscr{A}}$ are obvious, we write $U(\mathscr{A})$. Let $D(\mathscr{A}, M)$ be the K-module (under pointwise addition and scalar multiplication) of derivations from $\mathscr{A}$ to M. $\nu \in \mathrm{Hom}_{U(\mathscr{A})}(M_1, M_2)$ induces $D(\mathscr{A}, \nu) \in \mathrm{Hom}_K(D(\mathscr{A}, M_1), D(\mathscr{A}, M_2))$ in the obvious fashion. For further details of these objects see Jacobson [16].

Regarding $U(\mathscr{A})$ as the free $\mathscr{A}$-bimodule on one generator, we define, for $u \in U(\mathscr{A}), f_u\colon U(\mathscr{A}) \to U(\mathscr{A})$ such that $1_{U(\mathscr{A})}f_u = u$. $D(\mathscr{A}, U(\mathscr{A}))$ is a left $U(\mathscr{A})$-module under the multiplication $ud = dD(\mathscr{A}, f_u)$.

DEFINITION. An *inner derivation functor* is an epimorphism preserving subfunctor of $D(\mathscr{A}, \ )$.

For example, suppose $\mathscr{A}$ is Jordan. Define $J(\mathscr{A}, M)$ to be the K-module generated by all mappings of the form $\sum_i [R_{a_i} R_{m_i}]$ where $a_i \in \mathscr{A}$ and $m_i \in M$. Then $J(\mathscr{A}, M) \subseteqq D(\mathscr{A}, M)$ and J is an inner derivation functor.

THEOREM 1. *There is a one-to-one correspondance between the set of inner derivation functors and the set of left $U(\mathscr{A})$ submodules of $D(\mathscr{A}, U(\mathscr{A}))$.*

Proof. If $J(\mathscr{A}, \ ) \subseteqq D(\mathscr{A}, \ )$ is an inner derivation functor, define $\theta(J) = J(\mathscr{A}, U(\mathscr{A}))$. We need to define an inverse $\psi = \theta^{-1}$. Let $\Lambda \subseteqq D(\mathscr{A}, U(\mathscr{A}))$ be a sub-$U(\mathscr{A})$ module. If $M = \sum_{i \in \Gamma} \oplus U(\mathscr{A})$, define $J(\mathscr{A}, M) = \sum_{i \in \Gamma} \oplus \Lambda_i$, where $\Lambda_i \simeq \Lambda$ for all i. If M is any unital right $U(\mathscr{A})$-module, let X_M be the free unital right $U(\mathscr{A})$-module on the set M. Let Ω_M be the composite $\sum_{m \in M} \oplus \Lambda_m = J(\mathscr{A}, X_M) \xrightarrow{i} \sum_{m \in M} \oplus D(\mathscr{A}, X_m) = D(\mathscr{A}, X_M) \xrightarrow{D(\mathscr{A}, \Pi)} D(\mathscr{A}, M)$, where Π is the canonical projection $\Pi\colon X_M \to M$. Define $J(\mathscr{A}, M) = \text{image } \Omega_M$.