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A NOTE ON COMMUTATIVE INJECTIVE RINGS

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The purpose of this note is to determine necessary and sufficient conditions for a commutative injective ring to be a product of local rings. This reduces the study of such rings to the study of local rings, since a product of rings is injective if each factor is.

Throught R will be a ring with unit, J will denote its Jacobson radical, and if M is R module $E(M)$ will denote its injective hull. A ring R is right (sesp. left) self injective if it is injective as a right (resp. left) module over itself. One may easily verify that if $R = \pi R_i$, then R is right self injective if and only if each R_i is. Given an injective ring of a given type one would therefore like to realize it as a product of simpler rings. One example of a right self injective ring is the full ring of linear transformations on a vector space over a division ring. Faith [2] determined all rings which are products of full linear rings in the following:

THEOREM A. *The following are equivalent*:
(1) *R is a product of left full linear rings.*
(2) *R is a right self injective semiprime ring with large socle.*

If R is commutative the above theorem characterizes those rings which are products of fields. Our purpose here is to determine when a commutative injective ring may be written as a product of local rings. We prove two theorems:

THEOREM 2. *If R is a commutative injective ring with large socle then R is a product of local rings.*

THEOREM 6. *Let R be a commutative injective ring, then R is a product of local rings if and only if R/J has large socle and if $x \in R$, and x^1 denotes the right annihilator of x then $x^1 + J/J$ is not large in R/J.*

We will make use of the following easily available theorem [4]:

THEOREM B. *If R is injective then*
(1) *R/J is a regular ring*;
(2) *Idempotents left modulo J*;
(3) $J = \{r \in R \mid r^{\perp}$ *is large*$\}$.