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EXTENSIONS OF CONTINUOUS AFFINE FUNCTIONS

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Conditions are given for a closed face F of a compact convex set X to have the property that if $f \in A(F), g_1, \ldots, g_m \in A(X)$, and f dominates each g_i on F then f can be extended to $g \in A(X)$ where g dominates each g_i on X.

Let X be a compact convex set in a Hausdorff locally convex space. We identify X in the standard fashion with the set of positive elements of norm one in $A(X)^*$ (weak*-topology), where $A(X)$ is the ordered Banach space (sup-norm) of continuous affine functions on X. A face of X is a convex subset which contains the endpoints of every open line segment in X which it intersects. It is known (for example [2]) that every continuous affine function on a closed face F of X admits a continuous affine extension to all of X if and only if the linear span, $\langle F \rangle$, of F is weak* closed in $A(X)^*$. If additional conditions of a geometric nature on F and X are made then much more can be said about the type of extensions which are possible. For example if X is a Choquet simplex (in which case $\langle F \rangle$ is weak* closed whenever F is), a theorem of Edwards [3] states that
(*) if $\{f_i\}_{i=1}^m, \{g_j\}_{j=1}^n \in A(X)$ and $f \in A(F)$ such that

$$f_i|_F \leqq f \leqq g_j|_F \qquad (i = 1, \cdots, m; j = 1, \cdots n)$$

then there is an extension $g \in A(X)$ of f such that

$$f_i \leqq g \leqq g_j \qquad (i = 1, \cdots, m; j = 1, \cdots, n) .$$

This extension property is quite strong in the sense that it in fact characterizes simplexes among the compact convex sets.

One can ask under what conditions on F and X the following weaker extension property holds:
(**) if $\{f_i\}_{i=1}^m \in A(X)$ and $f \in A(F)$ such that

$$f_i|_F \leqq f \qquad (i = 1, \cdots, m)$$

then there is an extension $g \in A(X)$ of f such that

$$f_i \leqq g .$$

Closed faces which possess property (**) are termed *strongly archimedean* by Alfsen [1] (see also Størmer [5] for the origin of the terminology). In [2] we give conditions on F such that (**) holds for functions f_i, f identically zero on F. This implies in particular that F is (within a G_δ set) a peak-face of X. We give here a some-