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ON THE BERGMAN INTEGRAL OPERATOR FOR AN ELLIPTIC PARTIAL DIFFERENTIAL EQUATION WITH A SINGULAR COEFFICIENT

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Let $P_2(f)$ be Bergman's integral operator of the second kind. In this paper it is shown (1) $P_2(f)$ can be uniformly approximated by a linear combination of particular solutions; (2) $P_2(f)$ can be analytically continued; (3) $P_2(f)$ admits singular points if f is meromorphic.

In the study of functions of one complex variable one derives various relations between properties of the coefficients a_ν of the series development

$$(1)\qquad f(z) = \sum_{\nu=0}^{\infty} a_\nu z^\nu ,$$

of the function $f(z)$ and various properties of $f(z)$ in the large, such as the location and character of the singularities, growth of the function, etc. The method of integral operators enables one to generalize these theorems to the theory of linear partial differential equations

$$(2)\qquad L(\psi) = \frac{\partial^2 \psi}{\partial z \partial z^*} + A_1(z, z^*)\psi_z + A_2(z, z^*)\psi_{z^*} + A_3(z, z^*)\psi = 0 ;$$

$4(\partial^2\psi \backslash \partial z \partial z^*) = \Delta_1 \psi = (\partial^2 \psi \backslash \partial \lambda^2 + \partial^2 \psi \backslash \partial \theta^2)$, z, z^* are complex variables, $\lambda = z + z^* \backslash 2$, $\theta = z - z^* \backslash 2i$, $A_\nu(z, z^*)$, $\nu = 1, 2, 3$, are regular functions of z and z^* in a sufficiently large domain. The situation changes in the case when the A_ν admit singularities. In this paper we consider the equation

$$(3)\qquad L(\psi) = \Delta_1 \psi + 4F(\lambda)\psi \equiv 0 ,$$

where $F(s) = s^{-3}(a_0 + a_1 s + a_2 s^2 + \cdots + a_n s^n + \cdots)$, $s = (-\lambda)^{2/3}$, $a_0 = 5\backslash 144$, $a_1 = 0$, while the a_n, $n \geqq 2$ are such that $\overline{\lim}_{n\to\infty} |a_n|^{1/n} = 0$.

The integral operator

$$(4)\qquad \psi(z, z^*) \equiv P_2(f) = \int_l E(z, z^*, t) f\Big(\frac{z}{2}(1 - t^2)\Big)\frac{dt}{\sqrt{1 - t^2}}$$

(where l is some rectifiable Jordan path in the upper complex t-plane connecting the points -1 and 1), transforming analytic functions $f(z)$ in the neighborhood of the origin into solutions of (3), has been introduced and investigated by S. Bergman in [1, 2, 5, 6], see also