PACIFIC JOURNAL OF MATHEMATICS
Vol. 35, No. 3, 1970

ON MEARLY COMMUTATIVE DEGREE ONE ALGEBRAS

JOHN D. ARRISON AND MICHAEL RICH

The main result in this paper establishes that there do not exist nodal algebras A satisfying the conditions:
(I) $x(xy) + (yx)x = 2(xy)x$
(II) $(xy)x - x(yx)$ **is in** N**, the set of nilpotent elements of** A **over any field** F **whose characteristic is not two.**

Recall that a finite dimensional, power-associative algebra A with identity 1 over a field F is called a nodal algebra if every x in A is of the form $x = \alpha 1 + n$ with α in F and n nilpotent, and if the set N of nilpotent elements of A does not form a subalgebra of A. Following the convention laid down in [5] we call any ring satisfying (I) a nearly commutative ring.

In a recent paper [4] one of the authors has established the results given here if the field F has characteristic zero. In that paper the theorem of Albert [1] that there do not exist commutative, power-associative nodal algebras over fields of characteristic zero was used extensively. Recently, Oehmke [3] proved the same result if the field has characteristic $P \neq 2$. This result of Oehmke's will be used throughout this paper.

The known class of nodal algebras over fields of characteristic P are the truncated polynomial algebras of Kokoris [2] which are flexible. Our results show that if nearly commutative nodal algebras exist over fields of characteristic P they will not fall into the class of Kokoris algebras. In [5] one of the authors has shown that there do not exist nearly commutative nodal algebras over fields of characteristic zero.

Let A be a nearly commutative nodal algebra over a field F whose characteristic is $P \neq 2$. Then A^+ is a commutative, power-associative algebra over F. Therefore by [3] N^+ is a subalgebra of A^+. In particular, N is a subspace of A. The nilindex of A is defined to be the least positive integer k such that $n^k = 0$ for every n in N.

LEMMA 1. *There do not exist any nearly commutative nodal algebras whose nilindex is two over any field of characteristic* $P \neq 2$.

Proof. Let A be such an algebra. Then since $z^2 = 0$ for every z in N and N is a subspace of A we may linearize to get $xy = -yx$ for all x, y in N. Let $xy = \alpha 1 + n$, $yx = -\alpha 1 - n$. It suffices to