PACIFIC JOURNAL OF MATHEMATICS
Vol. 36, No. 1, 1971

TOPISMS AND INDUCED NON-ASSOCIATIVE SYSTEMS

KENNETH BACLAWSKI AND KENNETH M. KAPP

A triple of bijections $(\gamma;\ \alpha,\ \beta)$ from one binary system onto another is called an *isotopism* if the maps transfer the multiplication: $(xy)^\gamma = x^\alpha y^\beta$. These maps have been used by Albert, Bruck and others in the study of quasigroups and loops. A natural generalization of isotopism is given to a triple of maps called a topism, which transfers just the multiplication, as above, from one binary system into another. It will be shown, (1.7), that a topism from one quasigroup into another is an isotopism if and only if any one of the maps is one-to-one and onto.

Two maps, α, β on a binary system $(A,\ \cdot\ )$ induce a new binary operation, $\circ$, defined in the natural fashion $x \circ y = x^\alpha y^\beta$. The relationship between topic imbeddings and induced systems is studied in § 2. It is shown, (2.3), that one groupoid is topically imbeddable in another precisely when it is isomorphic to a subgroupoid of an induced groupoid of the second. Thus, (2.7), two quasigroups are isotopic if and only if one is isomorphic to an induced groupoid of the other.

Finally, the imbeddings of nonassociative binary systems into semigroups and groups are considered. It is shown, (3.1) and (3.2), that groupoids can be imbedded as ideals in semigroups. However, it is seen (3.4) that a groupoid with identity is topically imbeddable in a group precisely when the groupoid is isomorphic to a subsemigroup, with identity, of the given group. From this a generalization, (3.6), of Albert's Theorem that a loop is isotopic to a group if and only if they are isomorphic is obtained.

1. Preliminaries; topic imbedding.

DEFINITION 1.1. Let $(A,\ \cdot\ )$ and $(B,\ \circ\ )$ be two groupoids. An ordered triple $(\gamma;\ \alpha,\ \beta)$ of maps $\gamma,\ \alpha,\ \beta$ from A into B will be called a *topism* from A into B if $(x \cdot y)^\gamma = x^\alpha \circ y^\beta$ for all $x,\ y \in A$.

2. An *isotopy* from A onto B is a topism in which all three maps are one-to-one and onto.

3. A topism $(\gamma;\ \alpha,\ \beta)$ is said to be one-to-one or onto when γ is one-to-one or onto respectively.