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INTERSECTIONS OF NILPOTENT HALL SUBGROUPS

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A family $\mathscr{H}$ of subgroups of a finite group G is said to *satisfy* (*property*) B^* if whenever $U = H_1 \cap \cdots \cap H_r$ is a representation of U as intersection of elements of $\mathscr{H}$ of minimal length r, then $r \leqq 2$. The aim of this paper is to prove

THEOREM 1. Let H be a nilpotent Hall π-subgroup of a group G and assume that if $H_1, H_2 \in S_\pi(G)$ then $H_1 \cap H_2 \lhd H_1$. Then $S_\pi(G)$ satisfies B^*.

All groups in this work are finite. A family $\mathscr{H}$ of subgroups of a group G will be said *to satisfy* (*property*) B if there exist H_1 and H_2 in $\mathscr{H}$ such that

$$H_1 \cap H_2 = \bigcap \{H \mid H \in \mathscr{H}\} .$$

We will denote by $S_p(G)$ the family of Sylow p-subgroups of G and the (possibly empty) family of Hall π-subgroups of G will be denoted by $S_\pi(G)$. It was shown by Brodkey [1] that if G possesses an Abelian Sylow p-subgroup, then $S_p(G)$ satisfies B. Itô has shown in [3] that if G is of odd order, hence solvable by [2], then $S_p(G)$ satisfies B for all primes. He has also shown that if G is solvable, then $S_p(G)$ satisfies B in several other cases.

As indicated above, we will consider here a more restrictive condition B^* on families of subgroups of G. It follows from our main result, Theorem 1, that even the property B^* is satisfied by $S_\pi(G)$, whenever G possesses an Abelian or Hamiltonian (i.e., Dedekind) Hall π-subgroup. Theorem 1 yields the following

COROLLARY 1. *Let H be a nilpotent Hall subgroup of the group G and suppose that the index $[H\colon H \cap H^x]$ is prime for all $x \in G - N_G(H)$. Then either $H \lhd G$ or for all x, $y \in G$ such that $xy^{-1} \notin N_G(H)$ we have*

$$H^x \cap H^y = B \lhd G$$

and $[H\colon B] = p$, a prime. B is independent of x and y.

2. Generalizations. As a matter of fact, we will prove a more general result than Theorem 1. We will say that a group N *satisfies*