PACIFIC JOURNAL OF MATHEMATICS
Vol. 36, No. 3, 1971

ON THE NUMBER OF NON-ALMOST ISOMORPHIC MODELS OF T IN A POWER

SAHARON SHELAH

Let T be a first order theory. Two models are almost isomorphic if they are elementarily equivalent in the language $L_{\infty,\omega}$. We investigate the number of non almost-isomorphic models of T of power λ as a function of λ, $I(T,\lambda)$. We prove $\mu > \lambda \geqq |T|$, $I(T,\lambda) \leqq \lambda$ implies $I(T,\mu) \leqq I(T,\lambda)$. In fact, we generalize the downward Skolem-Lowenheim theorem for infinitary languages. Th. (1, 4, 5).

Let L be a set of predicates with finite number of places and sufficient number of variables. (We assume there are no function symbols in L for simplicity only.) $|L|$ will denote the number of predicates in L plus $\aleph_0$. Models will be denoted by M, N. The set of elements of M will be $|M|$, the cardinality of a set A by $|A|$ and so the cardinality of M by $\|M\|$. Unless specified otherwise, every model is an L-model. Cardinals will be denoted by $\lambda, \mu, \kappa, \chi$ ordinals i, j, α, β. T will denote a theory, i.e., set of sentences. We define $\mu^{(\lambda)} = \sum_{\kappa<\lambda} \mu^\kappa$. For cardinals λ, μ we define the language $L(\lambda, \mu)$ i.e., a set of formulas. This set is defined as the well known first-order language where we adjoin to its operations conjunction and disjunction on a set of $<\lambda$ formulas (i.e., $\bigwedge_{i\in I}\phi_i$, where $|I| < \lambda$) and existential or universal quantifications over a sequence of $<\mu$ variables. $L^*(\lambda, \mu)$ will be defined as $L(\lambda, \mu)$ where in addition we permit quantification of the form

$$[(\forall \bar{x}^1)(\exists \bar{y}^1) \cdots (\forall \bar{x}^n)(\exists \bar{y}^n) \cdots]_{n<\omega}$$

if

$$|\{x_0^1, x_1^1, \cdots, y_0^1, y_1^1, \cdots, x_0^n \cdots\}| < \mu \; .$$

$RL^*(\lambda, \mu)$ will denote the subset of $L^*(\lambda, \mu)$ consisting of the formulas Φ of $L^*(\lambda, \mu)$ such that for every subformula ϕ of Φ, if $\phi = [(\forall \bar{x}^1)(\exists \bar{y}^1) \cdots]\psi$, then $\models \phi \leftrightarrow \neg[(\exists \bar{x}^1)(\forall \bar{y}^1) \cdots] \neg \psi$. Clearly $RL^*(\lambda, \mu) \supset L(\lambda, \mu)$. K will denote any of those languages. Satisfaction (i.e., if $\phi = \phi(\bar{x})$, and $\bar{a}$ is a sequence from $|M|$, then $M \models \phi[\bar{a}]$) is defined naturally. (See Hanf [2] and Henkin [3].) The only nontotally trivial case is

$$\psi(\bar{z}) = [(\forall \bar{x}^0)(\exists \bar{y}^0)(\forall \bar{x}^1)(\exists \bar{y}^1) \cdots]\phi(\bar{z}, \bar{x}^0, \bar{x}^1, \cdots, \bar{y}^0, \bar{y}^1 \cdots) \; .$$

$M \models \psi[\bar{a}]$ if and only if there are functions $f_i^n(\bar{x}^0, \cdots, \bar{x}^n)$ such that for every sequence $\bar{a}^0, \bar{a}^1, \cdots$ from M, $M \models \phi[\bar{a}, \bar{a}^0, \bar{a}^1, \cdots, \bar{b}^0, \bar{b}^1, \cdots]$ where $\bar{b}^n = \langle \cdots, f_i^n(\bar{a}^0, \bar{a}^1, \cdots, \bar{a}^n), \cdots \rangle$. For a sentence ψ, $\models \psi$ if for