PACIFIC JOURNAL OF MATHEMATICS
Vol. 36, No. 3, 1971

NONTANGENTIAL HOMOTOPY EQUIVALENCES

VICTOR A. BELFI

The purpose of this paper is to apply surgery techniques in a simple, geometric way to construct manifolds which are nontangentially homotopy equivalent to certain π-manifolds. Applying this construction to an H-manifold of the appropriate type yields an infinite collection of mutually nonhomeomorphic H-manifolds, all nontangentially homotopy equivalent to the given one.

The theorem proved is the following: If N^{4k} is a smooth, closed, orientable π-manifold and L^m is a smooth, closed, simply connected π-manifold, there is a countable collection of smooth, closed manifolds $\{M_i\}$ satisfying (1) no M_i is a π-manifold, (2) each M_i is homotopy equivalent but not homeomorphic to $N \times L$, (3) M_i is not homeomorphic to M_j if $i \neq j$.

1. Construction of the surgery problem. In [2] Milnor describes a $(2k-1)$-connected, bounded π-manifold of dimension $4k$ and Hirzebruch index 8 $(k \geqq 2)$. This manifold, which we denote by Y^{4k}, is obtained by plumbing together 8 copies of the tangent disk bundle of S^{2k} according to a certain scheme. This implies that Y has the homotopy type of a bouquent of eight $2k$-spheres. The only other property of Y which we shall need is that ∂Y is a homotopy sphere. Let r be the order of ∂Y^{4k} in the group of homotopy spheres bP_{4k} [3] and take W^{4k} to be the r-fold connected sum along the boundary of Y^{4k}. By the choice of r, ∂W is diffeomorphic to S^{4k-1}. Attaching a $4k$-disk to W by a diffeomorphism along the boundary, we obtain a closed, smooth manifold $\hat{W}$, which is $(2k-1)$-connected and has index $8r$. By the Hirzebruch index theorem $\hat{W}$ is not a π-manifold, but is almost parallelizable.

Define $f: W^{4k} \rightarrow D^{4k}$ by the identity on the boundary, stretching a collar of ∂W over D^{4k}, and sending the remainder of W to a point. This gives a degree 1 map $f: (W, \partial W) \rightarrow (D^{4k}, \partial D^{4k})$ which is tangential since both W and D^{4k} are π-manifolds. f is already a homotopy equivalence on the boundary, so we have a surgery problem in the bounded case. The connectedness of W implies that f is already an isomorphism in homology below the middle dimension. However the kernel of f_* in dimension $2k$ is $\underbrace{Z \oplus \cdots \oplus Z}_{8r}$ and the index of the kernel is the index of W which is $8r$. Thus it is not possible to complete the surgery.

But if L^m is a closed, smooth, simply connected π-manifold, the surgery problem $f \times 1_L: W \times L \rightarrow D^{4k} \times L$ does have a solution. To