PACIFIC JOURNAL OF MATHEMATICS
Vol. 36, No. 3, 1971

ON DOMINATED EXTENSIONS IN LINEAR SUBSPACES OF $\mathscr{C}_C(X)$

E. M. ALFSEN AND B. HIRSBERG

The main result is the following: Given a closed linear subspace A of $\mathscr{C}_C(X)$ where X is compact Hausdorff and A contains constants and separates points, and let F be a compact subset of the Choquet boundary $\partial_A X$ with the property that the restriction to F of every A-orthogonal boundary measure remains orthogonal. If $a_0 \in A|_F$ and $a_0 \leqq \Psi|_F$ for some strictly positive A-superharmonic function Ψ, then a_0 can be extended to a function $a \in A$ such that $a \leqq \Psi$ on all of X. It is shown how this result is related to various known dominated extension-and peak set-theorems for linear spaces and algebras. In particular, it is shown how it generalizes the Bishop-Rudin-Carleson Theorem.

The aim of this paper is to study extensions within a given linear subspace A of $\mathscr{C}_C(X)$ of functions defined on a compact subset of the Choquet boundary $\partial_A X$, in such a way that the extended function remains dominated by a given A-superharmonic function Ψ. (Precise definitions follow). Our main result is the possibility of such extensions for all functions in $A|_F$ provided F satisfies the crucial requirement that the restriction to F of every orthogonal boundary measure shall remain orthogonal (Theorem 4.5). Taking $\Psi \equiv 1$ in this theorem we obtain that F has the norm preserving extension property (Corollary 4.6). This was first stated by Björk [5] for a real linear subspace A of $\mathscr{C}_R(X)$ and for a metrizable X. A geometric proof of the latter result was given by Bai Andersen [3]. In fact, he derived it from a general property of split faces of compact convex sets, which he proved by a modification of an inductive construction devised by Pelczynski for the study of simultaneous extensions within $\mathscr{C}_R(X)$ [12]. Our treatment of the more general extension property proceeds along the same lines as Bai Andersen's work. It depends strongly upon the geometry of the state space of A, and Bai Andersen's construction is applied at an essential point in the proof. Note however, that this is no mere translation of real arguments. The presence of complex orthogonal measures seems to present a basically new situation. Applying arguments similar to those indicated above, we obtain a general peak set-and peak point criterion (Theorem 5.4 and Corollary 5.5) of which the latter has been proved for real spaces by Björk [6]. In § 6 (Theorem 6.1) it is shown how the Bishop-Rudin-Carleson Theorem follows from the general extension theorem mentioned above. In § 7 we assume that A is a sup-norm algebra over X and study the inter-