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ON IDEALS IN Ω_*^U

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The objective of these notes is to study the relations between and the structure of various ideals that occur in the study of the complex bordism homology functor and actions of abelian p-groups on closed weakly complex manifolds.

Let us fix a prime integer p and denote by $K(Z_p, n)$ an Eilenberg-MacLane space of type (Z_p, n). Let

$$f: S^n \longrightarrow K(Z_p, n)$$

represent a generator of $\pi_n(K(Z_p, n)) = Z_p$ and set

$$\sigma_n = [S^n, f] \in \Omega_n^U(K(Z_p, n)) .$$

We propose to study the annihilator ideal, $A(\sigma_n) \subset \Omega_*^U$, of the U-bordism class σ_n. To this end let us denote by K the ideal which is the kernel of the natural map

$$\Phi_{(p)}: \Omega_*^U \longrightarrow H_*(BU; Z_p) .$$

Thus the elements of K are those U-manifolds all of whose Chern numbers are congruent to zero mod p. The structure of the ideal K is known as a consequence of the work of Milnor [8]. Namely, for each nonnegative integer i, there exists a "Milnor" manifold V^{2p^i-2}, of dimension $2p^i - 2$, such that

$$K = ([V^0], [V^{2p-2}], \cdots) .$$

The first elementary fact concerning the ideal $A(\sigma_n)$ is the inclusion $A(\sigma_n) \subset K$. Thus it makes sense to inquire into which of the Milnor manifolds $[V^0], [V^{2p-2}], \cdots,$ actually lie in $A(\sigma_n)$. One of our objectives in this note is to establish the following two results.

THEOREM A: $A(\sigma_n) \ni p, [V^{2p-2}], \cdots, [V^{2p^{n-1}-2}]$.

THEOREM B: $A(\sigma_n) \not\ni [V^{2p^n-2}], \cdots, [V^{2p^{n+m}-2}], \cdots$.

Thus we determine completely which of the Milnor manifolds annihilate the class σ_n. While this does not determine the structure of $A(\sigma_n)$ it is a step towards that goal. The study of annihilator ideals of spherical bordism classes has been a recurrent theme throughout the investigations [5], [6], [10], [11] of the complex bordism of finite complexes. The ideal $A(\sigma_n)$ is in an appropriate sense a universal example of such an ideal.