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# A FACTORIZATION THEOREM FOR ANALYTIC FUNCTIONS OPERATING IN A BANACH ALGEBRA

PHILIP C. CURTIS, JR. AND HENRIK STETKAER

**Cohen's factorization-theorem asserts that if the Banach algebra $\mathfrak{A}$ has a left approximate identity, then each $y \in \mathfrak{A}$ may be written $y = xz$, $x, z \in \mathfrak{A}$. The vector $x$ may be chosen to be bounded by some fixed constant and $z$ may be chosen arbitrarily close to $y$. In this setting the theorem below asserts that if $F$ is a holomorphic function defined on a sufficiently large disc about $\zeta = 1$, and satisfying $F(1) = 1$, then each $y \in \mathfrak{A}$ may be written $y = F(x)z$, where $x, z \in \mathfrak{A}$. Again $x$ may be chosen to be bounded by some fixed constant and $z$ may be chosen close to $y$.**

We state and prove our result using the terminology of [2]. The proof is an elaboration of the proof of Theorem 2.2 of [2]. In what follows $X$ is a complex Banach space, $\mathscr{E} = \{E_\alpha\}$ is a uniformly bounded subset of $B(X)$ which we may assume to be directed and which satisfies $\lim_\alpha E_\alpha E = E$ for each $E \in \mathscr{E}$. Convergence is in the norm topology of $B(X)$. Let

$$Y = \{x \in X\colon \lim_\alpha E_\alpha x = x\} ,$$

and let $\mathfrak{A}$ be the closed subalgebra of $B(X)$ generated by $\mathscr{E}$.

For further extensions of Cohen's theorem we refer the reader to Chapter 8 of [3].

THEOREM. *Let $F$ be a holomorphic complex-valued function with $F(1) = 1$, defined on a neighbourhood of $\{z \in \boldsymbol{C} \mid |z - 1| \leqq M\}$, $M > 1$, where $\|E - I\| \leqq M$ for all $E \in \mathscr{E}$.*

*Then to every $y \in Y$ and $\delta > 0$ there exist $z \in Y$ and $U \in \mathfrak{A}$ such that*

$$y = F(U)z \text{ and } \|y - z\| < \delta .$$

*If furthermore $F$ has no zeros in the open interval $]0, 1[$, then $U$ may for some $a \in ]0, 1[$ be written in the form*

$$U = \sum_1^\infty a(1 - a)^{k-1} E_k ,$$

*where $E_k \in \mathscr{E}$ for $k = 1, 2, \cdots$.*