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A STUDY OF CERTAIN SEQUENCE SPACES OF MADDOX AND A GENERALIZATION OF A THEOREM OF IYER

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In this paper we examine the Köthe-Toeplitz reflexivity of certain sequence spaces and we characterize some classes of matrix transformations defined on them. The results are used to prove a generalization of a theorem by V. G. Iyer, concerning the equivalence of the notions of strong and weak convergence on the space of all integral functions, and also to generalize some theorems by Ch. Rao.

Let X, Y be two nonempty subsets of the space s of all complex sequences and $A = (a_{nk})$ an infinite matrix of complex numbers $a_{nk}(n, k = 1, 2, \cdots)$. For every $x = (x_k) \in X$ and every integer n we write

$$A_n(x) = \sum_k a_{nk}x_k ,$$

where the sum without limits is always taken from $k = 1$ to $k = \infty$. The sequence $Ax = (A_n(x))$, if it exists, is called the transformation of x by the matrix A. We say that $A \in (X, Y)$ if and only if $Ax \in Y$ whenever $x \in X$.

Throughout the paper, unless otherwise indicated, $p = (p_k)$ will denote a sequence of strictly positive numbers (not necessarily bounded in general). The following classes of sequences were defined by Maddox [4] (see also Simons [10], Nakano [8]):

$$l(p) = \left\{x: \sum_k |x_k|^{p_k} < \infty\right\} ,$$

$$l_\infty(p) = \left\{x: \sup_k |x_k|^{p_k} < \infty\right\} ,$$

$$c(p) = \left\{x: |x_k - l|^{p_k} \longrightarrow 0 \text{ for some } l\right\} ,$$

$$c_0(p) = \left\{x: |x_k|^{p_k} \longrightarrow 0\right\} .$$

When all the terms of (p_k) are constant and all equal to $p > 0$ we have $l(p) = l_p$, $l_\infty(p) = l_\infty$, $c(p) = c$, and $c_0(p) = c_0$, where l_p, l_∞, c, c_0, are respectively the spaces of p-summable, bounded, convergent and null sequences. It is easy to see that $l_\infty(p) = l_\infty$ if and only if $0 < \inf\, p_k \leqq \sup p_k < \infty$ and similarly for $c_0(p) = c_0$, $c(p) = c$ (see [4]). It was shown in [4], [5], [6], that the sets $l(p)$, $l_\infty(p)$, $c(p)$ and $c_0(p)$ are linear spaces under coordinatewise addition and scalar multiplication