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LIE HOMOMORPHISMS OF OPERATOR ALGEBRAS

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A mapping $\phi: M \to N$ between *-algebras M, N which is *-linear, and which preserves the Lie bracket $[X, Y] = XY - YX$ of elements X, Y in M is called a Lie *-homomorphism or just a Lie homomorphism. The main result of this paper states that if $\phi: A \to B$ is a uniformly continuous Lie *-homomorphism of the C*-algebra A onto the C^*-algebra B then there exists a central projection D in the weak closure of B such that modulo a center-valued *-linear map which annihilates brackets, $D\phi$ is a *-homomorphism and $(I - D)\phi$ is the negative of a *anti-homomorphism.

Previously we showed that if M is a *factor* then so is N and in this case $\phi = \sigma + \lambda$ where σ is a *-isomorphism or the negative of *a* *-anti-isomorphism of M onto N and λ is a *-linear functional which annihilates brackets in M. This result parallels the algebraic theorems of L. Hua and W. S. Martindale.

The main techniques used in this paper are the algebraic techniques of Martindale [8], [9], and Herstein [3]. Adaptations of them allow us to characterize Lie *-isomorphisms between von Neumann algebras and also ultra-weakly ($= UW$) closed Lie *-ideals which contain the center. For a complete exposition concerning Lie structures on associative algebras we recommend Herstein [4]. We wish to thank Professor H. A. Dye for many invaluable conversations during the preparation of this paper.

1. Preliminaries and notation.

We denote by $\mathscr{L}(H)$ the ring of all linear operators $T: H \to H$, H a complex Hilbert space with inner product $(.\,,\,.)$, which are bounded in the norm $\|T\| = \sup_{\|x\| \leq 1} \|Tx\|$. With this norm, $\mathscr{L}(H)$ is a Banach algebra with identity the identity operator I. In addition to the uniform topology on $\mathscr{L}(H)$ we shall be concerned with (1) the weakest topology making the linear functionals $T \to (Tx, y)$ continuous for all $x, y \in H$, called the weak (operator) topology and (2) the weakest topology making the linear functionals $T \to \sum_{n=1}^{\infty} (Tx_n, y_n)$ continuous for all sequences $\{x_n\}$ $\{y_n\}$ such that $\sum_{n=1}^{\infty} \|x_n\|^2$, $\sum_{n=1}^{\infty} \|y_n\|^2 < \infty$ called the ultra-weak (operator) topology.

To each operator $T \in \mathscr{L}(H)$ there corresponds an operator $T^* \in \mathscr{L}(H)$, called the adjoint of T, defined by $(Tx, y) = (x, T^*y)$ for all $x, y \in H$. If $T = T^*$, T is called self-adjoint; if T is selfadjoint and $T = T^2$, T is called a projection; if $TT^* = T^*T = I$, T is called unitary.

A C^*-algebra, M, is a subalgebra of $\mathscr{L}(H)$ which is closed in the