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UNIQUELY REPRESENTABLE SEMIGROUPS ON THE TWO-CELL

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A semigroup S is said to be uniquely representable in terms of two subsets X and Y of S if $X \cdot Y = Y \cdot X = S$, $x_1y_1 = x_2y_2$ is a nonzero element of S implies $x_1 = x_2$ and $y_1 = y_2$, and $y_1x_1 = y_2x_2$ is a nonzero element of S implies $y_1 = y_2$ and $x_1 = x_2$ for x_1, $x_2 \in X$ and $y_1, y_2 \in Y$. A semigroup S is said to be uniquely divisible if for each $s \in S$ and every positive integer n there exists a unique $z \in S$ such that $z^n = s$. Theorem. If S is a uniquely divisible semigroup on the two-cell with the set of idempotents of S being a zero for S and an identity for S, then S is uniquely representable in terms of X and Y where X and Y are iseomorphic copies of the usual unit interval and the boundary of S equals X union Y. Corollary. If S is a uniquely divisible semigroup on the two-cell and if S has only two idempotents, a zero and an identity, then the nonzero elements of S form a cancellative semigroup.

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The primary purpose of this paper is to show that if S is a uniquely divisible semigroup on two-cell with the set of idempotents of S being a zero for S and an identity for S, then S is uniquely representable in terms of X and Y where X and Y are iseomorphic copies of the usual unit interval and the boundary of S equals X union Y. As a corollary to this theorem we shall prove a conjecture of D. R. Brown, that if S is a uniquely divisible semigroup on the two-cell and if S has only two idempotents, a zero and an identity, then the nonzero elements of S form a cancellative subsemigroup of S.

NOTATION. Throughout S will be a uniquely divisible semigroup on the two-cell with $E(S)$ (the set of idempotents of S) $= \{0, 1\}$ where 0 is the zero for S and 1 is the identity for S. It is well known that the boundary of S is the union of two usual threads X and Y with $X \cap Y = \{0, 1\}$ and $S = X \cdot Y = Y \cdot X$. Intervals containing x will represent segments of X and intervals with y shall stand for segments of Y. For a positive integer n, $s^{1/n}$ will denote the unique nth root of s in S.