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MATRIX CHARACTERRIZATIONS OF CIRCULAR-ARC GRAPHS

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A graph G is a circular-arc graph if there is a one-to-one correspondence between the vertices of G and a family of arcs on a circle such that two distinct vertices are adjacent when the corresponding arcs intersect. Circular-arc graphs are characterized as graphs whose adjacency matrix has the quasi-circular 1's property. Two interesting subclasses of circular-arc graphs are also discussed proper circular-arc graphs and graphs whose augmented adjacency matrix has the circular 1's property.

Given a finite family S of nonempty sets, the *intersection graph* $G(S)$ has vertices corresponding to the sets of S and two distinct vertices of $G(S)$ are adjacent if and only if the corresponding sets of S intersect. Suppose the graph G is isomorphic to $G(S)$ (i.e., there is a one-to-one correspondence between vertices which preserves adjacency). Then S is called an *intersection model* for G. If S is a family of arcs on a circle, G is called a *circular-arc graph*. See the example in Figure 1. If, in addition, no arc of S contains another arc, G is called a *proper circular-arc graph* (the graph in Figure 1a is not a proper circular-arc graph). Interval graphs and proper interval graphs are analogously defined. Lekkerkerker and Boland [10] have given a structure theorem for interval graphs (in the spirit of Kuratowski's famous characterization of planar graphs). Fulkerson and Gross [4] characterized interval graphs as graphs whose dominant clique-vertex incidence matrix ("dominant" means maximal) has the *consecutive* 1'*s property for colums*, that is, the rows of this matrix can be permuted so that the 1's appear consecutively in each column. They also gave [4] an efficient algorithm to test whether a $(0, 1)$-matrix has the consecutive 1's property for columns. For other characterizations of interval graphs, see Gilmore and Hoffman [5] and Lekkerkerker and Boland [10]. The study of interval graphs was motivated by the central role they played in some work by Benzer [1, 2] concering the molecular substructure of genes. More recently, interval graphs have been applied to problems in archeology [8] and ecology [3]. Proper interval graphs have been characterized by Roberts [11, 12] with a structure theorem and as graphs whose augmented adjacency matrix (defined below) has the consecutive 1's property for columns. He showed that they were equivalent to the indifference graphs of mathematical psychology.