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COHOMOLOGY OF GROUP GERMS AND LIE ALGEBRAS

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Let π be a continuous representation of a Lie group G in a finite dimensional real vector space V. Denote by $H_\square(G, V)$ the cohomology with empty supports in the sense of Sze-tsen Hu. If L is the Lie algebra of G, π induces an L-module structure on V and there is the associated cohomology $H(L, V)$ of Chevalley-Eilenberg. Our main result is the construction of an isomorphism $H_\square(G, V) \simeq H(L, V)$.

This is preceded by a closer analysis of $H_\square(G, V)$. It is clear from the definition that to know $H_\square(G, V)$, it suffices to know an arbitrary neighbourhood of 1 in G and its action on V. The totality of neighbourhoods of 1 in G may be regarded as an object of a more fine nature than a local group; we call it a group germ. More precisely, a group germ is defined as a group object in the category Γ of topological germs [18]. The Eilenberg-MacLane definition [3] of the cohomology of an abstract group is carried over from the category of sets to Γ (i.e., from groups to group germs). Thus for any group germs g, a, where a is abelian, and any g-action on a, we have cohomology groups $H(g, a)$. It turns out that $H_\square(G, V) \simeq H(g, a)$ for a suitable choice of g and a, in all dimensions > 1. To cope with dim 0 and 1 it seems convenient to introduce the concept of an action of a group germ g on an abelian topological group A and associate with this a cohomology $H(g, A)$. This is only a slight modification of the previous $H(g, a)$, so that both cohomologies coincide in dimensions >1 and $H^1(g, A)$ is a quotient of $H^1(g, a)$, if a is suitably related to A. ($H^0(g, A)$ is the subgroup of g-stable elements of A and $H^0(g, a)$ is always trivial). One now has $H_\square(G, V) \simeq H(g, V)$ in all dimensions, for a group germ g corresponding to G.

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1. Group germs. Let T be the category of pointed topological spaces. For $A, B \in T$ write $A \simeq B$ if and only if there is a $C \in T$ which is an open subspace of both A and B. Denote by $[A]$ the equivalence class of A. For morphisms $f: A \to B, f': A' \to B'$ in T write $f \simeq f'$ if and only if $A \cong A'$, $B \simeq B'$ and there is a $C \in T$ which is an open subspace of both A and A' such that $f \mid C = f' \mid C$. Denote the equivalence class of $f: A \to B$ by $[f]: [A] \to [B]$. There is now precisely