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GROUP RINGS SATISFYING A POLYNOMIAL IDENTITY II

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In an earlier paper we obtained necessary and sufficient conditions for the group ring $K[G]$ to satisfy a polynomial identity. In this paper we obtain similar conditions for a twisted group ring $K^t[G]$ to satisfy a polynomial identity. We also consider the possibility of $K[G]$ having a polynomial part.

1. Twisted group rings. Let K be a field and let G be a (not necessarily finite) group. We let $K^t[G]$ denote a twisted group ring of G over K. That is $K^t[G]$ is an associative K-algebra with basis $\{\bar{x} \mid x \in G\}$ and with multiplication defined by

$$\bar{x}\bar{y} = \gamma(x, y)\overline{xy}\ , \qquad \gamma(x, y) \in K - \{0\}\ .$$

The associativity condition is equivalent to $\bar{x}(\bar{y}\bar{z}) = (\bar{x}\bar{y})\bar{z}$ for all $x, y, z \in G$ and this is equivalent to

$$\gamma(x, yz)\gamma(y, z) = \gamma(x, y)\gamma(xy, z)\ .$$

We call the function $\gamma: G \times G \to K - \{0\}$ the factor system of $K^t[G]$. If $\gamma(x, y) = 1$ for all $x, y \in G$ then $K^t[G]$ is in fact the ordinary group ring $K[G]$. In this section we offer necessary and sufficient conditions for $K^t[G]$ to satisfy a polynomial identity. The proof follows the one for $K[G]$ given in [3] and we only indicate the suitable modifications needed. The following is Lemma 1.1 of [2].

LEMMA 1.1. *If $x \in G$, then in $K^t[G]$ we have*

(i) $1 = \gamma(1, 1)^{-1}\,\bar{1}$

(ii) $\bar{x}^{-1} = \gamma(x, x^{-1})^{-1}\gamma(1, 1)^{-1}\,\overline{x^{-1}}$
$\qquad = \gamma(x^{-1}, x)^{-1}\gamma(1, 1)^{-1}\,\overline{x^{-1}}$.

PROPOSITION 1.2. *Suppose $K^t[G]$ satisfies a polynomial identity of degree n and set $k = (n\,!)^2$. Then G has a characteristic subgroup G_0 such that $[G: G_0] \leqq (k + 1)\,!$ and such that for all $x \in G_0$*

$$[G: C_G(x)] \leqq k^{4^{(k+1)!}}\ .$$

Proof. This is the twisted analog of Corollary 3.5 of [3]. We consider § 3 of [3] and observe that each of the prerequisite results for that corollary also has a twisted analog.