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STRICTLY CYCLIC OPERATOR ALGEBRAS

ALAN LAMBERT

This paper is concerned with the structure of abelian algebras $\mathscr{A}$ of operators on Hilbert space $\mathscr{H}$ such that $\mathscr{A}x = \mathscr{H}$ for some vector x in H. It is shown that if a transitive algebra $\mathscr{T}$ contains such an algebra then $\mathscr{T}$ is dense in the weak topology on $\mathscr{L}(\mathscr{H})$. It is also shown that when an algebra of this type is semi-simple then it is a reflexive operator algebra. The algebras investigated have the property that every densely defined linear trans-formation commuting with the algebra is bounded.

Let $\mathscr{H}$ be a complex Hilbert space and let $\mathscr{L}(\mathscr{H})$ be the algebra of all bounded linear operators on $\mathscr{H}$. The study of subalgebras of $\mathscr{L}(\mathscr{H})$ has primarily dealt with self-adjoint algebras. The literature on non-self-adjoint subalgebras of $\mathscr{L}(\mathscr{H})$ is far less complete. This paper is concerned with a class of non-self-adjoint subalgebras, the strictly cyclic abelian subalgebras. The first application of these algebras will be to the theory of transitive algebras. A subalgebra $\mathscr{T}$ of $\mathscr{L}(\mathscr{H})$ is *transitive* if the only closed subspace of $\mathscr{H}$ invariant for every operator in $\mathscr{T}$ are $\mathscr{H}$ and $\{0\}$. W. B. Arveson showed that a knowledge of the (possibly) unbounded linear transformations commuting with a transitive algebra $\mathscr{T}$ can be used to decide if $\mathscr{T}$ is dense in the weak operator topology on $\mathscr{L}(\mathscr{H})$ (it is not kown if every transitive algebra of operators on an infinite dimensional Hilbert space must be weakly dense in $\mathscr{L}(\mathscr{H})$).

Arveson also proved that every transitive algebra containing a maximal abelian self-adjoint algebra is weakly dense in $\mathscr{L}(\mathscr{H})$. E. Nordgren, H. Radjavi, and P. Rosenthal used Arveson's techniques to show that if $\mathscr{H}$ is separable, then every transitive algebra of operators containing a certain type of weighted shift must be dense in $\mathscr{L}(\mathscr{H})$. It is shown that every transitive algebra containing a strictly cyclic abelian algebra is weakly dense in $\mathscr{L}(\mathscr{H})$. It has been shown that the weakly closed algebras generated by certain weighted shifts are strictly cyclic. This class of shifts properly contains the class of shifts mentioned above. In particular, several examples of shifts generating strictly cyclic algebras are neither compact nor quasi-nilpotent.

In § 3 we develop some tests for strict cyclicity of abelian algebras. In § 5 we show that certain stictly cyclic abelian algebras are unitarily equivalent to multiplication operator algebras on functional Hilbert spaces (Theorem 5.1), and are examples of reflexive operator