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AMITSUR COHOMOLOGY OF ALGEBRAIC NUMBER RINGS

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A bound is given for the order of the Amitsur cohomology group $H^i(S/R, U)$ corresponding to an extension $R \subset S$ of rings of algebraic integers. The effect of inflation on the Chase-Rosenberg exact sequence involving Amitsur cohomology and split Brauer groups is also studied.

It is well known [9] that global class field theory, together with some results of Auslander-Goldman [2], leads to the determination of the split Brauer group $B(S/R)$ corresponding to an extension $R \subset S$ of rings of algebraic integers. Although the Amitsur cohomology group $H^2(S/R, U)$ is related to $B(S/R)$ by the Chase-Rosenberg exact sequence [5], $H^2(S/R, U)$ has only been computed in case $R = \boldsymbol{Z}$ and S is quadratic (see [10] and [8]). In this note we prove $H^i(S/R, U)$ is finite for all i (Corollary 2.2) and, as in [8], derive further information in case $i = 2$ by applying inflation to the Chase-Rosenberg sequence.

Throughout the paper, rings and algebras are commutative with unit elements and algebra homomorphisms are unitary. We assume familiarity with the Amitsur cohomology, Brauer group and Pic functors (see [4], [2] and [3] respectively) and with spectral sequences.

2. Finiteness of chomology.

The aim of this section is to establish a bound for the order of Amitsur cohomology groups in the unit functor U for extensions of rings of algebraic integers.

PROPOSITION 2.1. *Let R be a Dedekind domain with quotient field K, S the integral closure of R in an n-dimensional separable field extension L of K, and T the integral closure of R in a normal closure F of L/K. Assume $U(T)$ can be generated by m elements. Then, for all $i \geqq 0$, the Amitsur cohomology group $H^i(S/R, U)$ is finite, with order at most $n^{(m(n!)^i)}$.*

Proof. We first observe that the cochain group $C^{i-1}(S/R, U) = U(S \otimes_R \cdots \otimes_R S) = U(S^i)$ is finitely generated for all $i \geqq 1$. By a standard argument [12, Chapter V, Theorem 7], S and T are module-finite faithful R-projectives such that $S \otimes_R K \cong L$; hence the R-rank of S is n. Let $G = \mathrm{gal}(F/K)$. Flatness provides injective R-algebra homomorphisms $S^i \rightarrow T^i \rightarrow F^i$; composition with the canonical isomor-