PACIFIC JOURNAL OF MATHEMATICS
Vol. 40, No. 1, 1972

DEFINING RELATIONS FOR CERTAIN INTEGRALLY PARAMETERIZED CHEVALLEY GROUPS

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For each faithful finite dimensional irreducible representation R of a finite dimensional simple Lie algebra L over the complex field, this paper treats the integrally parameterized subgroup G_Z of the Chevalley Group G over the rational field Q. For L of type A_l, D_l, or E_l, Lie algebraic methods are used to extend a result of J. Nielson on $SL(3, Z)$ to obtain a finite set of defining relations for G_Z. Similar relations augmented by defining relations for $G_Z(B_2)$ are shown to define G_Z when L is of type B_l, C_l, or F_4. (The relations for $G_Z(B_2)$ are not listed here.)

Defining relations for the n-dimensional group of lattice transformations have been given by W. Magnus in [4]. His method easily yields relations for the group $SL(n, Z)$ respectively $PSL(n, Z)$ isomorphic to the universal respectively adjoint group G_Z for L of type A_{n-1}. H. Klingen [2] has proven the existence of a finite set of defining relations for $Sp(2n, Z)$, which is essentially the group G_Z for L of type C_n. Hence, the defining relations in § 2 extend Magnus' result to G_Z of types D_l and E_l and Klingen's result to G_Z of types B_l and F_4.

It might be helpful to the reader to note that a displayed equation is referred to by a symbol in parentheses, e.g., "(3.1)" or "(B)" and a theorem, lemma, or corollary is referred to by its title and a number without parentheses, e.g., "Lemma 3.1".

2. Statement of results. Let R be a faithful finite dimensional irreducible representation of a finite dimensional simple Lie algebra L over the complex field C, and let Σ be the set of nonzero roots of L with respect to some Cartan subalgebra. L has a *Chevalley basis* $\{X_\rho, H_\rho: \rho \in \Sigma\}$ as defined in [1, p. 24, Th. 1] or [9, p. 6, Th. 1]. The L module V associated with R contains a lattice M which is invariant under the action of the Chevalley basis. If M is properly chosen and K is an arbitrary field, the automorphism $x_\rho(t) = \exp tR(X_\rho)$ on $V_K = K \otimes_Z M$ can be defined for each ρ in Σ and t in K. The group G_K generated by all of these automorphisms is the Chevalley group over the field K of type L corresponding to the representation R. G_K is the *adjoint* respectively *universal* Chevalley group if R is the adjoint respectively universal representation of L. (See [9, pp. 42-45].)

We will be concerned with the *rational Chevalley group* G_Q (henceforth denoted by G) and its subgroup, the *integrally parameterized Chevalley group* G_Z generated by the $x_\rho(t)$ with ρ in Σ and t in Z.