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SUMMABILITY AND FOURIER ANALYSIS

GEORGE BRAUER

An integration on βN, the Stone-Cech compactification of the natural numbers N, is defined such that if s is a bounded sequence and ϕ is a summation method evaluating s to σ, $\int s d\phi = \sigma$. The Fourier transform $\hat{\phi}$ of a summation method ϕ is defined as a linear functional on a space of test functions analytic in the unit disc: if

$$f(z) = \sum_{n=0}^{\infty} \hat{f}(n) z^n, \ |z| < 1, \text{ then } \hat{\phi}(f) = \int \hat{f}(n) d\phi .$$

A functional which agrees with the Fourier transform of a regular summation method must annihilate the Hardy space H_1. Our space of test functions is often the space M_p of functions $f = \sum \hat{f}(n) z^n$, analytic in the unit disc, such that

$$||f||_{M_p} = \lim\ \sup[(1-r)\int_0^{2\pi} |f(r^{1/p}e^{i\theta})|^p \ d\theta/2\pi]^{1/p}$$

is finite for some $p > 1$. A functional L which is well defined on a space M_p for some $p \geqq 2$ such that $L(1/(1-z)) = 1$ agrees with the Fourier transform of a summation method which is slightly stronger than convergence.

Let $s = \{s_n\}$ be an infinite sequence of complex numbers, that is, a continuous function on the discrete additive semigroup of natural numbers N. The sequence s has a continuous extension s^β to βN, the Stone-Cech compactification of N (s^β takes the value ∞ if s is unbounded). Throughout the paper, the symbol βZ denotes the Stone-Cech compactification of the space Z, and the continuous extension of a function f defined on Z to βZ will be denoted by f^β; for a description of the Stone-Cech compactification we refer the reader to [2, pp. 82-93]. We impose the norm

$$\begin{aligned} ||s|| &= \lim \sup |s_n| \\ &= LUB\,|s^\beta(\gamma)| \ , \quad \gamma \in \beta N - N \end{aligned}$$

on the space m_0 of bounded sequences. Thus m_0 is isometric to $C(\beta N - N)$, the ring of continuous complex functions on $\beta N - N$; sequences differing by a null sequence are identified in m_0.

Let ϕ denote a summation method-that is, a linear functional on a subspace of m_0. We assume that the ϕ-transform of every sequence s to which ϕ is applicable is either a continuous function on N or else a continuous function on the half open unit interval I: $[0, 1)$.