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FIXED POINT AND COINCIDENCE SETS OF BICONNECTED MULTIFUNCTIONS ON TREES

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Various properties of fixed point sets of monotone single-valued mappings on dendrites have been studied by L. E. Ward, Jr. and the author. Some of them are extended here to coincidences of two mappings between two trees, and to biconnected multifunctions. Examples are given which show that others are no longer true in these cases.

1. Introduction.

We know several properties of fixed point sets of monotone mappings onto dendrites. A theorem by G. E. Schweigert [5] and L. E. Ward, Jr. [6] states that every monotone mapping of a dendrite onto itself which leaves one end point fixed also leaves at least one other point fixed. Further results are contained in [3], where e.g., monotone mappings which leave all end points or all but finitely many end points fixed are investigated. It is shown in [3] that the fixed point set contains all branch points of order $\geqq n$ if it contains all but n of the end points.

The theorem by Schweigert and Ward has been extended in [4] in two directions: from fixed points to coincidences of two mappings, and from monotone single-valued mappings to biconnected multifunctions. The purpose of the present paper is an attempt to extend results from [3] in a similar way. But this is not entirely possible. In the Main Theorem (see § 3) we only prove that points of order $\geqq n + 3$ rather than order $\geqq n$ are invariant, and examples in § 4 show that this statement cannot be sharpened to include points of order $\leqq n + 2$. Neither can another result from [3], namely the non-existence of a fixed point set consisting of two points of order two for monotone surjections of dendrites, be extended to coincidences or to biconnected multifunctions, as can be seen from two examples given at the end.

A dendrite is a metric tree. The conclusions of this paper are valid for trees, as a metric is not needed in the proofs.

2. Trees and multifunctions.

A tree is a continuum (i.e., compact connected Hausdorff space) in which every pair of points is separated by a third one. We use the partial order structure of trees which was developed by L. E. Ward, Jr. [6, 7]. It is obtained by selecting an arbitrary point $r \in T$ as root, and defining $x \leqq y$ if $x = r$, $x = y$, or x separates r and y. Let