PACIFIC JOURNAL OF MATHEMATICS
Vol. 40, No. 3, 1972

SIMPLE POINTS IN PSEUDOLINE ARRANGEMENTS

L. M. KELLY AND R. ROTTENBERG

A finite collection of simple closed curves in the real projective plane each two of which have exactly one point in common at which point they "cross" is called a pseudoline arrangement. Such arrangements seem to have been first systematically studied by Levi. Recently B. Grünbaum has called attention to the desirability for a better understanding of the differences as well as the similarities in the behavior of arrangements of lines and the arrangements of pseudolines.

Among other things, Grünbaum asks if every pseudoline arrangement, not all curves of which intersect in a single point, must have a simple vertex (a point on exactly two of the curves of the arrangement) as is the case in line arrangements. In fact Kelly and Moser have shown that, in general, an n-line arrangement in an ordered projective plane has at least $3/7n$ simple vertices. It is shown here that their reasoning can be nearly dualized to prove the analogous result for pseudoline arrangements.

However we make heavy use of a lemma of Levi proved in [4]. Examples show that it is possible to realize pseudoline arrangements which are not combinatorially equivalent to any line arrangement.

2. Preliminaries, definitions, notation.

DEFINITION 2.1. A finite collection, F, of simple closed curves in the real projective plane, each two of which have exactly one point in common, at which point they cross, is called a *pseudoline arrangement*. Each curve is called a *pseudoline* or, where there is no danger of confusion merely a line. The set of points of intersection of the lines of F is denoted $I(F)$ and each point of $I(F)$ is a *vertex* of F. If exactly two lines of F pass through a vertex it is *simple* or *ordinary*.

LEMMA OF LEVI. *If F is a pseudoline arrangement in the real projective plane $\mathscr{P}$ and A and B any two points of $\mathscr{P}$, then there exists a simple closed curve, g, containing A and B such that $F \cup \{g\}$ is a pseudoline arrangement.*

By successive applications of this lemma we obtain a pseudoline arrangement containing F such that each two points of $I(F)$ are on exactly one line of the arrangement. Each line of F is intersected by the lines of this extended arrangement in a finite number of points