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ON A QUESTION OF TARSKI AND A MAXIMAL THEOREM OF KUREPA

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Let *PI* be the statement, "Every Boolean algebra has a prime ideal."; let *SPI* be the statement, "Every infinite set algebra has a nonprincipal prime ideal."; let *K* be the statement, "Every family of sets includes a maximal subfamily of pairwise incomparable (by the inclusion relation) sets.". A model is exhibited of set theory without regularity in which *SPI* and *K* hold, *PI* fails. Furthermore all finitary versions of the axiom of choice fail in this model and the model contains a set which is infinite Dedekind finite in the sense of the model. A proof is given that the local version of *SPI* implies the axiom of choice for families of finite sets in *ZF*.

1. Introduction and statement of results. Before 1963, the only method available for constructing interesting models of Zermelo-Fraenkel set theory, (*ZF*), was the Fraenkel-Mostowski-Specker method which produced models in which the axiom of regularity failed. In 1963, Cohen presented his method for producing models in which regularity holds. Questions concerning the axiom of choice, (*AC*), can be considered in both types of models, unlike questions such as the continuum hypothesis which are only interesting in Cohen models. Although an independence result established by the Cohen method is stronger than the same result established by the *FMS* method, this latter method remains of interest for a number of reasons as Lévy points out in [2]. Two such reasons are as follows: (1) There are independence results which hold in set theory without regularity but do not hold in set theory with regularity. (2) The proof of a given independence may be more easily established by means of an *FMS* model. Such a proof may in turn give a clue to the stronger result. In this connection David Pincus has recently indicated a method of converting *FMS* models to Cohen models which gives the stronger result in many cases. In this paper we will give an example of each eventuality mentioned above. Consider the following maximal principles:

PI. Every Boolean algebra has a prime ideal.

SPI. Every infinite set algebra has a nonprincipal prime ideal.

K. (Kurepa's principle). Every family of sets includes a maximal subfamily of pairwise incomparable sets. (Two sets are comparable if one is a subset of the other.)

We shall exhibit an *FMS* model in which *K* holds but C_2 (the