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STOCHASTIC INTEGRALS IN ABSTRACT WIENER SPACE

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Let $W(t, \omega)$ be the Wiener process on an abstract Wiener space (i, H, B) corresponding to the canonical normal distributions on H. Stochastic integrals $\int_0^t \xi(s, \omega) dW(s, \omega)$ and $\int_0^t (\zeta(s, \omega), dW(s, \omega))$ are defined for non-anticipating transformations ξ with values in $\mathscr{B}(B, B)$ such that $(\xi(t, \omega) - I)(B) \subset B^*$ and ζ with values in H. Suppose $X(t, \omega) = x_0 + \int_0^t \xi(s, \omega) dW(s, \omega) + \int_0^t \sigma(s, \omega) ds$, where σ is a non-anticipating transformation with values in H. Let $f(t, x)$ be a continuous function on $R \times B$, continuously twice differentiable in the H-directions with $D^2 f(t, x) \in \mathscr{B}_1(H, H)$ for the x variable and once differentiable for the t variable. Then $f(t, X(t, \omega)) = f(0, x_0) + \int_0^t (\xi^*(s, \omega) Df(s, X(s, \omega), dW(s, \omega)) + \int_0^t \{\partial f / \partial s(s, X(s, \omega)) + \langle Df(s, X(s, \omega)), \sigma(s, \omega) \rangle + \frac{1}{2} \operatorname{trace}[\xi^*(s, \omega) D^2 f(s, X(s, \omega)) \xi(s, \omega)]\} ds$, where $\langle\ ,\ \rangle$ is the inner product of H. Under certain assumptions on A and σ it is shown that the stochastic integral equation $X(t, \omega) = x_0 + \int_0^t A(X(s, \omega)) dW(s, \omega) + \int_0^t \sigma(X(s, \omega)) ds$ has a unique solution. This solution is a homogeneous strong Markov process.

1. Introduction. The notion of stochastic integral introduced by K. Ito [5; 8] is well-known nowadays [10]. Its generalization to infinite dimensional space has been investigated and used for the study of differential equations of diffusion type. See, for instance, [1] and [2]. The purpose of this paper is to define and study stochastic integrals with respect to standard Wiener process in an abstract Wiener space [3; 4]. We will generalize Ito's formula [7] and study the stochastic integral equation. We remark that our work is quite different from that of [1] and closely related to that of [2]. However, we have a more general space and weaker assumptions than [2].

2. Abstract Wiener space. In this section we give a brief review of the notion of an abstract Wiener space and establish notation at the same time. Let H be a real separable Hilbert space with norm and inner product denoted by $|\cdot|$ and $\langle\ ,\ \rangle$ respectively. Let $\mu_t (t > 0)$ be the cylinder set measure on H defined by $\mu_t(C) = (2\pi t)^{-n/2} \int_D \exp\{-|x|^2/2t\} dx$, where $C = P^{-1}(D)$, D is a Borel set in the image of an n-dimensional projection P in H and dx is Lebesgue measure in PH. A *measurable norm* on H is a norm $\|\cdot\|$ on H with