PACIFIC JOURNAL OF MATHEMATICS
Vol. 41, No. 2, 1972

ON A REPRESENTATION OF A STRONGLY HARMONIC RING BY SHEAVES

KWANGIL KOH

A ring R is strongly harmonic provided that if M_1, M_2 are a pair of distinct maximal modular ideals of R, then there exist ideals $\mathscr{A}$ and $\mathscr{B}$ such that $\mathscr{A} \not\subseteq M_1$, $\mathscr{B} \not\subseteq M_2$ and $\mathscr{A}\mathscr{B} = 0$. Let $\mathscr{M}(R)$ be the maximal modular ideal space of R. If $M \in \mathscr{M}(R)$, let $O(M) = \{r \in R \mid$ for some $y \notin M$, $rxy = 0$ for every $x \in R\}$. Define $\mathscr{R}(R) = \bigcup \{R/O(M) \mid M \in \mathscr{M}(R)\}$. If R is a strongly harmonic ring with 1, then R is isomorphic to the ring of global sections of the sheaf of local rings $\mathscr{R}(R)$ over $\mathscr{M}(R)$. Let $\Gamma(\mathscr{M}(R), \mathscr{R}(R))$ be the ring of global sections of $\mathscr{R}(R)$ over $\mathscr{M}(R)$. For every unitary (right) R-module A, let $A_M = \{a \in A \mid aRx = 0$ for some $x \notin M)$ and let $\tilde{A} = \bigcup\{A/A_M \mid M \in \mathscr{M}(R)\}$. Define $\hat{a}(M) = a + A_M$ and $\hat{r}(M) = r + O(M)$ for every $a \in A$, $r \in R$ and $m \in \mathscr{M}(R)$. Then the mapping ξ_A: $a \mapsto \hat{a}$ is a semi-linear isomorphism of A onto $\Gamma(\mathscr{M}(R)), \mathscr{R}(R))$—module $\Gamma(\mathscr{M}(R), \tilde{A})$ in the sense that ξ_A is a group isomorphism satisfying $\xi_A(ar) = \hat{a}\hat{r}$ for every $a \in A$ and $r \in R$.

1. If R is a ring with 1, R is called *harmonic* (or regular) if the maximal modular ideal space, say $\mathscr{M}(R)$, with the hull-kernel topology, is a Hausdorff space (refer [5]). A ring R is *strongly harmonic* provided that for any pair of distinct maximal modular ideals M_1, M_2 there exist ideals $\mathscr{A}$, $\mathscr{B}$ in R such that $\mathscr{A} \not\subseteq M_1$, $\mathscr{B} \not\subseteq M_2$ and $\mathscr{A}\mathscr{B} = 0$. For any nonempty subset S of a ring R define $(S)^\perp = \{r \in R \mid sr = 0$ for every $s \in S\}$ and if $a \in R$ let aR_1 be the principal right ideal generated by a. If M is a prime ideal of a ring R let $O(M) = \{r \in R \mid (rR_1)^\perp \not\subseteq M\}$. An ideal $\mathscr{A}$ of a ring R is called *M-primary* for some maximal modular ideal M of R provided that $M/\mathscr{A}$ is the unique maximal modular ideal of $R/\mathscr{A}$ and if $\mathscr{A}'$ is an ideal of R such that $\mathscr{A}' \subseteq \mathscr{A}$ and $\mathscr{A}' \neq \mathscr{A}$ then $R/\mathscr{A}'$ is no longer a local ring (here by a local ring we mean a ring with the unique maximal modular ideal). The principal results in this paper are as follows: Let R be a ring such that if R/S is a local ring for some ideal S of R then R/S has a unit. Then R is strongly harmonic if and only if $O(M)$ is M-primary for every maximal modular ideal M of R. If R is a strongly harmonic ring with 1 then R is isomorphic to $\Gamma(\mathscr{M}(R), \mathscr{R}(R))$ the ring of global sections of the sheaf of local rings $\mathscr{R}(R) = \bigcup \{R/O(M) \mid M \in \mathscr{M}(R)\}$ over $\mathscr{M}(R)$ and if A is a unitary right R-module then the mapping ξ_A: $a \mapsto \hat{a}$ is a semi-linear isomorphism of A onto $\Gamma(\mathscr{M}(R), \mathscr{R}(R))$—