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A TECHNIQUE FOR THE DETECTION OF OSCILLATION OF SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

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An iterative procedure is used to determine the oscillatory properties of second order linear differential equations, using a repeated application of the Kummer-Liouville transformation.

1. The Kummer-Liouville transformation. We shall consider the oscillatory properties of the equation

$$(1) \qquad \begin{gathered}(a(t)x')' + c(t)x = 0 \qquad \left(' = \frac{d}{dt}\right)\\ a(t) \in C^1[t_0, \infty),\ a(t) > 0\ ,\\ c(t) \in C[t_0, \infty)\ .\end{gathered}$$

Let $\varphi(t) \in C^1[t_0, \infty)$, $\varphi'(t) > 0$, $\lim_{t\to\infty} \varphi(t) = \infty$,

$$\psi(t) \in C^2[t_0, \infty),\ \psi(t) \neq 0\ , \quad t_0 \leqq t < \infty\ .$$

It was shown by Kummer ([5], 1834) that the transformation $\tau = \varphi(t)$, $x(t) = \psi(t)y(\tau)$ transforms the equation (1) into an equation of the same form:

$$(1^a) \qquad (R(\tau)y'(\tau))' + Q(\tau)y(\tau) = 0\ .$$

The formulas for $R(\tau)$, $Q(\tau)$ are:

$$\begin{aligned}R(\tau(t)) &= a(t)\varphi'(t)\psi^2(t)\\ Q(\tau(t)) &= [(a(t)\psi'(t))' + c(t)\psi(t)[\varphi'(t)]^{-1}]\psi(t)\ ,\end{aligned}$$

(see [5], or the expository article [7]). Moreover, if $\varphi(t)$ is chosen to be

$$\varphi(t) = \int_{t_0}^{t} [a(\xi)\psi^2(\xi)]^{-1} d\xi\ ,$$

then the equation (1^a) assumes the form

$$(2) \qquad y''(\tau) + \sigma(\tau)y(\tau) = 0\ .$$

In the specific case when $\int_t^{\infty} [a(\xi)]^{-1} d\xi = \infty$, the choice $\psi(t) \equiv 1$ results in the formula