PACIFIC JOURNAL OF MATHEMATICS
Vol. 42, No. 1, 1972

STRUCTURE OF RIGHT SUBDIRECTLY IRREDUCIBLE RINGS II

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The object of this paper is to determine the structure and properties of right subdirectly irreducible rings which are either local or self-injective. The rings in the latter class form a special case of the so-called right *PF* rings. By employing the notion of Feller's *X*-rings, it is proved that right *PF* *X*-rings are finite direct sums of full matrix rings over self-injective right subdirectly irreducible rings. Thus, whether or not right *PF* *X*-rings are left *PF* depends on the answer to the same question for the more elementary case of self-injective right subdirectly irreducible rings.

For a discussion of artinian and noetherian *RSI* rings, see [2].

1. Notation and preliminaries.

All rings considered have an identity and all modules are unitary. A module M_R is R-subdirectly irreducible if the intersection of all nonzero submodules of M is nonzero, which will then be called the heart of M_R. A ring R is *RSI* (right subdirectly irreducible) if R_R is R-subdirectly irreducible. The heart H of a *RSI* ring R is a two sided ideal. These and some of the following definitions and observations are given in [2] and we rewrite them for completeness. We will always use the following notation is connection with a *RSI* ring R. $H =$ heart, $N = H^l = \{x \in R: xH = 0\}$, $D = \mathrm{Hom}_R(H_R, H_R)$, $\hat{R} =$ injective hull of R_R, $K = \mathrm{Hom}_R(\hat{R}, \hat{R})$ and $L = \{f \in K: \ker f \neq 0\}$. In addition, for a local ring R, J will always denote the unique maximal right ideal. A ring R will be termed self-injective if R_R is injective.

We state the following theorem showing the relationship between R, N, H, D, K, L which has been proved in [2, p. 319].

THEOREM 1.1. *If R is RSI, then R/N is isomorphic to a subring of the division ring D and $D \cong K/L$.*

In connection with $QF-1$ algebras, faithful indecomposable modules play an important role. In the following proposition we prove that a *RSI* ring has a unique faithful indecomposable injective module. In this respect, it may be remarked that an artinian semisimple ring which is not simple is an example of a ring for which faithful indecomposable injectives don't exist; while over the ring of integers, for each prime p, by using [12, p. 145, Th. 7] or otherwise,