PACIFIC JOURNAL OF MATHEMATCIS
Vol. 42, No. 2, 1972

ON THE ABSOLUTE HAUSDORFF SUMMABILITY OF A FOURIER SERIES

S. N. LAL AND SIYA RAM

In this paper a theorem on the absolute Hausdorff summability of a series associated with a Fourier series has been established. This theorem unifies and extends various known results.

1. Let μ_n be a sequence of real or complex numbers and write

$$\Delta^0\mu_n = \mu_n,\ \Delta^p\mu_n = \Delta^{p-1}\mu_n - \Delta^{p-1}\mu_{n+1}\ , \quad p \geqq 1\ .$$

If S_n denotes the sequence of partial sums of the series $\sum_{n=0}^{\infty} a_n$, the transformation

$$t_m = \sum_{n=0}^{m} \binom{m}{n}(\Delta^{m-n}\mu_n)S_n$$

defines the sequence $\{t_m\}$ of (H, μ) means or the Hausdorff means [3, 12] of the sequence $\{S_n\}$. The series $\sum a_n$ is said to be summable (H, μ) to the sum s if $\lim_{m\to\infty} t_m = s$ and is said to be absolutely summable (H, μ) or summable $|H, \mu|$ if

$$\sum_{n=1}^{\infty} |t_n - t_{n-1}| < C^1\ .$$

In order that (H, μ) should be a convergence preserving transformation it is necessary and sufficient that μ_n should be a moment constant, that is, there exists a function $\chi(x)$ of bounded variation in $0 \leqq x \leqq 1$, such that

$$\mu_n = \int_0^1 x^n d\chi(x)\ , \qquad n = 0, 1, 2, \cdots\ .$$

We may suppose without loss of generality that $\chi(0) = 0$. If also, $\chi(1) = 1$ and $\chi(+0) = \chi(0) = 0$, so that $\chi(x)$ is continuous at the origin, then μ_n is a regular moment constant and (H, μ) is a regular Hausdorff transformation [3]. It is known that the sequence to sequence Hausdorff transformation is absolute convergence preserving or absolutely regular if and only if it is a convergence preserving or regular transformation of the same type [4, 8, 9].

In the case in which

$$\chi(x) = 1 - (1 - x)^\delta\ , \quad \delta > 0\ ,$$

[1] Throughout the paper C denotes a positive constant not necessarily the same at each occurrence.