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A DECOMPOSITION THEOREM FOR BIADDITIVE PROCESSES

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This paper treats a class of stochastic processes called biadditive processes and gives a proof of a decomposition of their sample functions. Informally, a biadditive proces $X(s, t)$ is a process indexed by two time parameters whose "increments" over disjoint rectangles are independent. The increments of such a process are the second differences

$$X(s_2, t_2) - X(s_1, t_2) - X(s_2, t_1) + X(s_1, t_1)$$

where $s_1 < s_2$ and $t_1 < t_2$. The decomposition theorem states that every centered biadditive process is the sum of four independent biadditive processes: one with jumps in both variables, two with jumps in one variable and continuous in probability in the other, and a fourth process which is jointly continuous in probability.

This decomposition is similar to one for processes with independent increments and in the proofs of both results a major role is played by the theory of centralized sums of independent random variables.

More formally, let $P_1 = \{s_1, s_2, \cdots, s_n\}$ and $P_2 = \{t_1, t_2, \cdots, t_m\}$ be two partitions of $[0, s_n]$ and $[0, t_m]$ respectively. Define $P_1 \times P_2$ to be the corresponding partition of $[0, s_n] \times [0, t_m]$ into rectangles whose vertices are the (s_i, t_j)'s. Let Δ_{ij} denote the increment

$$\Delta_{ij} = X(s_{i+1}, t_{j+1}) - X(s_i, t_{j+1}) - X(s_{i+1}, t_j) + X(s_i, t_j)$$

over the rectangle with vertices (s_{i+1}, t_{j+1}), (s_i, t_{j+1}), (s_{i+1}, t_j) and (s_i, t_j). Then if the increments

$$\{\Delta_{ij} \colon i = 0, 1, \cdots, n-1, j = 0, 1, \cdots, m-1\}$$

corresponding to any partition $P_1 \times P_2$ are independent and if $X(s, 0) = 0 = X(0, t)$ for all s and t not less than zero, $X(s, t)$ is called biadditive.

It is easy to construct some examples of biadditive processes. For instance, if $\{Y_{ij}\}_{i,j=0}^{\infty}$ is a doubly infinite sequence of independent random variables, then it is easy to see that the process

$$X(s, t) = \sum_{i<s} \sum_{j<t} Y_{ij}$$

is biadditive. A nontrivial example of a biadditive process is obtained when the space C_2 of continuous functions of two variables on $[0, \infty) \times [0, \infty)$ is given the Wiener-Yeh measure and the process $X(s, t)$ is the