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COHOMOLOGY OF FINITELY PRESENTED GROUPS

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Let G be a finitely presented group, G' a finite quotient of G and K a field. Let G act on the group algebra $V = K[G']$ in the natural way. For a suitable choice of G' we obtain estimates on the dimension of $H^1(G, V)$ in terms of the presentation and then use these estimates to derive information about G.

If G is generated by n elements, of which m have finite orders $k_1, \cdots, k_m$, resp., and G has the presentation

$$\langle a_1, \cdots, a_n; a_1^{k_1}, \cdots, a_m^{k_m}, r_{m+1}, \cdots, r_{m+q}\rangle ,$$

then, in particular, we show that (a) the minimum number of generators of G is $\geqq n - q - \sum 1/k_i$; (b) if this lower bound is actually attained, then G is free, of this rank, and (c) G is infinite if $\sum 1/k_i \leqq n - q - 1$. The latter, together with a result of R. Fox, yields an algebraic proof that the group

$$\langle a_1, \cdots, a_m; a_1^{k_1}, \cdots, a_m^{k_m}, a_1 \cdots a_m\rangle$$

is infinite if $\sum 1/k_i \leqq m - 2$.

1. An exact sequence. Let G be a group with the presentation $\langle a_1, \cdots, a_n; r_1, r_2, \cdots\rangle$, i.e., $G = F/N$, where F is the free group on $\{a_1, \cdots, a_n\}$ and N is the normal subgroup generated by $\mathscr{R} = \{r_1, r_2, \cdots\}$. We denote by φ the homomorphism of group rings $\boldsymbol{Z}[F] \rightarrow \boldsymbol{Z}[G]$ which extends the natutral map $F \rightarrow F/N$, and by A_i the element φa_i of G.

Let ρ be a representation of G in Aut(V), where V is a finite-dimensional vector space over a field K. We shall be concerned with the first cohomology group $H^1(G, V)$, which is also a vector space over K in an obvious way. One knows that an arbitrary map $f: \{A_1, \cdots, A_n\} \rightarrow V$ extends to a 1-cocycle of G in V if and only if the 1-cocycle of F determined by $a_i \mapsto f(A_i)$ vanishes on the relators. More precisely, the following sequence is exact:

$$(*) \qquad 0 \longrightarrow Z^1(G, V) \xrightarrow{E} V^n \xrightarrow{D} V_1 \oplus V_2 \oplus \cdots .$$

Here, $Z^1(G, V)$ is the space of 1-cocycles, V^n is the direct sum of n copies of V, $V_i = V$ for each i, E is the map $f \mapsto (f(A_1), \cdots, f(A_n))$, D is the map

$$(u_1, \cdots, u_n) \longmapsto \left(\sum_j (\partial r_1/\partial a_j)u_j, \sum_j (\partial r_2/\partial a_j)u_j, \cdots\right) ,$$

and in the last term of the sequence there is one copy of V for each