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UNICOHERENT COMPACTIFICATIONS

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In this paper we give necessary and sufficient conditions for the Freudenthal compactification of a rimcompact, locally connected and connected Hausdorff space to be unicoherent. We give several necessary and sufficient conditions for a locally connected generalized continuum to have a unicoherent compactification and show that if such a space X has a unicoherent compactification, then γX is the smallest unicoherent compactification of X in the usual ordering of compactifications.

A connected topological space X is said to be *unicoherent* if, $H \cdot K$ is connected whenever $X = H + K$ where H and K are closed connected sets. A continuum is a compact connected metric space and a generalized continuum is a locally compact, connected, separable metric space. By a mapping we will always mean a continuous function. If B is a subset of a space X, the closure of B in X will be denoted by $\mathrm{cl}_X B$ and the boundary of B in X will be denoted by $\mathrm{Fr}_X B$. An open set (respectively, a closed set) of a space X will be called a γ-open (respectively, γ-closed) subset of X provided it has a compact boundary in X. A space is rimcompact (or semicompact) provided every point has arbitrarily small neighborhoods with compact boundaries. All compactifications considered here are Hausdorff.

In [7] K. Morita showed that for any rimcompact Hausdorff space X there exists a topologically unique compactification γX of X satisfying:

(a) For every point x of γX and every open set R of γX containing x there exists an open set V of γX containing x such that $V \subset R$ and $\mathrm{Fr}_{\gamma X} V \subset X$.

(b) Any two disjoint γ-closed subsets of X have disjoint closures in γX.

Furthermore if C is any compactification of X satisfying (a), there exists a mapping h of γX onto C such that $h|X$ is the identity map. The compactification γX of X is called the Freudenthal compactification of X after H. Freudenthal who first defined it [4].

DEFINITION. We say that a connected space X is γ-unicoherent if whenever $X = H + K$, where H and K are γ-closed and connected sets, $H \cdot K$ is connected.

THEOREM 1. *If X is a locally connected, connected, rimcompact Hausdorff space, then γX, the Freudenthal compactification of X, is*