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THE EVALUATION MAP AND *EHP* SEQUENCES

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Let $L(\Sigma B, X)$ be the space of maps from ΣB (the reduced suspension of B) to X with the compact-open topology, let $\ell\colon \Sigma B \to X$ and $L(\Sigma B, X; \ell)$ the path component of $L(\Sigma B, X)$ containing ℓ. For nice spaces the evaluation map ω: $L(\Sigma B, X, \ell) \to X$ defined by $\omega(f) = f(*)$ is a fibration and gives rise to a long exact sequence in homotopy. The purpose of this paper is to show that the boundary map in that long exact sequence can be given by a generalized Whitehead product and that the sequence generalizes the *EHP* sequence of G. W. Whitehead.

1. Preliminary definitions. All spaces are assumed to be CW complexes with base point at a vertex. Maps are base point preserving. The cartesian product $A \times B$ is assumed to be based at (a_0, b_0), the unit inverval, I, is based at 0, and quotient spaces are based at the image of the base point under the natural quotient map. Where the space is clear $*$ will denote the base point as well as the constant map with image at the base point.

We use the following notations. $L(A, B)$ will denote the space of maps from A to B with the compact-open topology and $L(A, B; \ell)$ the path component of $L(A, B)$ containing $\ell\colon A \to B$. $L_0(A, B)$ and $L_0(A, B; \ell)$ will denote the space of base point preserving maps in $L(A, B)$ and $L(A, B; \ell)$ respectively. Let $A \vee B$ and $A \# B$ denote the one point union and smash product respectively.

Since spaces are assumed to be CW complexes the smash product can be taken as $A \times B$ with $A \vee B$ identified with (a_0, b_0). $q\colon A \times B \to A \# B$ will denote the quotient map. Note that $S^{p+q} = \Sigma^p S^q = S^p \# S^q$, $\Sigma^p A = S^p \# A$, and $\Sigma(A \vee B) = \Sigma A \vee \Sigma B$.

Let $p_1, p_2\colon A \times B \to A \vee B$ be defined by $p_1(a, b) = a \vee b_0$ and $p_2(a, b) = a_0 \vee b$. Define $k\colon \Sigma(A \times B) \to \Sigma A \vee \Sigma B$ by $k = \Sigma p_1 + \Sigma p_2 - \Sigma p_1 - \Sigma p_2$. Since $k \mid \Sigma(A \vee B)$ homotopically trivial, by the homotopy extension property there is a map $k'\colon \Sigma(A \times B) \to \Sigma A \vee \Sigma B$, homotopic to k, such that $k' \mid \Sigma(A \vee B) = *$. k' then induces a map $\tilde{k}\colon \Sigma(A \# B) \to \Sigma A \vee \Sigma B$. Arkowitz [1] has shown that $[\tilde{k}]$ is uniquely determined by the requirement $k \cong \tilde{k} \circ \Sigma q$. The following definition is due to Arkowitz [1].

DEFINITION 1.1. For $\alpha = [f] \in [\Sigma A, X]$ and $\beta = [g] \in [\Sigma B, X]$, the *generalized Whiteheal product* $[\alpha, \beta]$ is defined by $[\alpha, \beta] = [(f \vee g) \circ \tilde{k}] \in [\Sigma(A \# B) X]$.