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FUNCTION ALGEBRAS OVER VALUED FIELDS

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In this paper we consider primarily algebras $F(T)$ of continuous funtions taking a topological space T into a complete nonarchimedean nontrivially valued field F. Some general properties of function algebras and topological algebras over valued fields are developed in §§1 and 2. Some principal results (Theorems 6 and 7) are analogs of theorems of Nachbin and Shirota, and Warner: Essentially that $F(T)$ with compact-open topology is F-barreled iff unbounded functions exist on closed noncompact subsets of T; and that full Fréchet algebras are realizable as function algebras $F(\mathscr{M})$ where $\mathscr{M}$ denotes the space of nontrivial continuous homomorphisms of the algebra.

Nachbin and Shirota's well-known result provides a necessary and sufficient condition for an algebra of realvalued continuous functions on a topological space to be barreled when it carries the compact-open topology. To develop an analog of Nachbin's theorem for F-valued functions, it is necessary to bypass the heavily real-number-oriented machinery on which his proof depends. We accomplish this in part by developing an ordering of the elements of a discretely valued field (Sec. 3, Def. 2) which serves to take the place of the usual ordering of the reals. We also consider a notion of "support" of a continuous F-valued linear functional on $F(T)$ (Sec. 3, Def. 3). The support notion is developed without measure theory or representation theorems for continuous linear functionals.

The results of the paper depend heavily on theorems proved by Ellis ([3]), Kaplansky ([7], [8]), and van Tiel ([14]), as well as the proofs of the major theorems as originally presented by Nachbin ([10]) and Warner ([15]) which provided the ideas for this line of approach.

Throughout the paper "algebra" (denoted by X or Y) includes the presence of an identity and commutativity. The underlying field F is assumed to be a complete nonarchimedean rank one nontrivially valued field. Unless otherwise stated, T denotes a 0-dimensional (a base for the topology consisting of closed and open sets exists) Hausdorff topological space and $F(T)$ the algebra of continuous functions from T into F with pointwise operations. The terms Banach space or Banach algebra are used throughout in the sense of [12].

1. Topological algebras over valued fields. In this section we discuss some basic properties of topological algebras over fields with valuation. We assume throughout that the underlying field F is a