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ABEL SUMMABILITY OF CONJUGATE INTEGRALS

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It is proved here that the conjugate Fourier-Stieltjes integral of a finite-valued Borel measure μ on Euclidean k-space, $k \geqq 1$, taken with respect to a Calderon-Zygmund kernel in Lip $\alpha, 0 < \alpha < 1$, is almost everywhere (with respect to Lebesgue measure) Abel summable to the conjugate function of μ taken with respect to the above mentioned kernel. This has been already established for $k \leqq 3$ and for k even.

We make the following assumptions: k is a positive integer; $0 < \alpha < 1$; $\Omega \in \operatorname{Lip} \alpha(S)$, where S denotes the $(k-1)$-sphere in k-dimensional Euclidean space E_k; $\int_S \Omega(y)dS(y) = 0$, where dS refers to the natural measure on S; and μ is a real Borel measure on E_k as defined in [3].

Let $K(x) = \Omega(x/|x|)|x|^{-k}$ for each nonzero x in E_k (we use $|x|$ for the usual norm and $x \cdot y$ for the usual dot product and dx for Lebesgue measure, all in E_k). For y in E_k, set $\hat{\mu}(y) = (2\pi)^{-k} \int_{E_k} e^{-ix \cdot y} d\mu(x)$ and

$$\hat{K}(y) = (2\pi)^{-k} \lim_{\substack{\varepsilon \to 0 \\ R \to \infty}} \int_{\varepsilon \leqq |x| \leqq R} e^{-ix \cdot y} K(x) dx .$$

It is known [5, p. 69] that $\hat{K}$ is bounded on E_k. We define, for $t > 0$ and x in E_k,

$$I_t(x) = (2\pi)^k \int_{E_k} e^{-t|y|} \hat{k}(y) \hat{\mu}(y) e^{ix \cdot y} dy .$$

We shall prove the

THEOREM. $$\lim_{t \to 0} \left[I_t(x) - \int_{|y-x|>t} K(x-y) d\mu(y) \right] = 0$$

except on a set of Lebesgue measure zero in E_k.

If $k = 1$, the theorem is classical (see [8, p. 103] for the essence of the matter). The case $k = 2$ and $1/2 < \alpha < 1$ was treated in [4]. The cases in which $0 < \alpha < 1$ and $k = 3$ or k is even were handled in [2]. Further references and motivation for the theorem are given in [2] and [4]. The proof given in the present paper covers all cases with $0 < \alpha < 1$ and $k \geqq 3$; modifications could be made in the proof to cover the cases $k = 2$, $0 < \alpha < 1$ and $k = 1$, but this seems pointless.